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# The role of $e^-$ -capture in neutrino-nucleosynthesis

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## Abstract

In the first stage of this paper, we perform detailed calculations of the electron capture cross sections on nuclei under laboratory conditions. We use the nuclear method known as proton-neutron quasi-particle random phase approximation (pn-QRPA). In the second stage, we translate the above mentioned  $e^-$ -capture cross sections to the stellar environment. As a concrete nuclear target we use the  $^{56}\text{Fe}$  and the  $^{66}\text{Zn}$  isotopes, which belong to the iron group nuclei and play prominent role in stellar nucleosynthesis: i) in pre-supernova phase and ii) in supernova phase.

**Keywords:** *Original Electron Capture, Stellar Electron Capture, Stellar Nucleosynthesis, Semi-leptonic charged current reactions*

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## Introduction

In the early stage of collapse (for densities lower than a few  $10^{10} \text{ gr/cm}^3$ ), the electron chemical potential is of the same order of magnitude as the nuclear  $Q$  value, and the  $e^-$ -capture cross-sections are sensitive to the details of GT strength distributions in daughter nuclei (at these densities, electrons are captured mostly on nuclei with mass number  $A \approx 60$ ) [1,2,3,4]. Various methods, used for calculating  $e^-$ -capture on nuclei during the collapse phase, have shown that this process produces neutrinos with rather low energies in contrast to the inelastic neutrino-nucleus reactions occurring in supernova [5,6]. These neutrinos escape the star carrying away energy and entropy from the core which is an effective cooling mechanism of the exploding massive star [7]. For higher densities and temperatures,  $e^-$ -capture occurs on heavier nuclei  $A > 60$  [2,3,4]. As a consequence, the nuclear composition is shifted to more neutron-rich and heavier nuclei which dominate the matter composition for densities larger than about  $10^{10} \text{ gr/cm}^3$  [4,7,8].

In this work, we perform detailed calculations of electron capture cross sections in pre-supernova conditions for  $^{56}\text{Fe}$  nuclear isotope and in supernova conditions for  $^{66}\text{Zn}$  using the pn-QRPA method. Our strategy in this work is, at first to perform extensive calculations of the transition rates for all the above mentioned nuclear processes assuming laboratory conditions, and then to translate these rates to the corresponding quantities within stellar environment through the use of an appropriate convolution procedure [1,4,8,9]. To this purpose, we assume that the energy distribution of the initial states of the parent nucleus under such conditions follow Maxwell-Boltzmann distribution [1,9].

## Theoretical Background

Electrons of energy  $E_e$  are captured by nuclei interacting weakly with them via  $W^-$  boson exchange as

$$(A, Z) + e^- \rightarrow (A, Z-1) + \nu_e \quad (1)$$

The outgoing  $\nu_e$  neutrino carries energy  $E_\nu$  while the daughter nucleus  $(A, Z-1)$  absorbs a part of the incident electron energy given by the difference between the initial  $E_i$  and the final  $E_f$  nuclear energies as  $E_\nu = E_f - E_i$ .

From the nuclear theory point of view, the main task is to calculate the cross sections of the reaction (1) which are based on the evaluation of the nuclear transition matrix elements between the initial  $|i\rangle$  and a final  $|f\rangle$  nuclear states of the form

$$\langle f | \widehat{H}_w | i \rangle = \frac{G}{\sqrt{2}} \ell^\mu \int d^3x e^{-i\mathbf{q}\cdot\mathbf{x}} \langle f | \widehat{\mathcal{T}}_\mu | i \rangle.$$

For reliable predictions of total cross sections, a consistent description of the structure of the ground state  $|J_i\rangle$  of the parent nucleus as well as of the multipole excitations  $|J_f\rangle$  of the daughter nucleus are required.

### Results

In this work we perform detailed cross section calculations for the electron capture on  $^{56}\text{Fe}$  and  $^{66}\text{Zn}$  isotopes on the basis of the pn-QRPA method.

The required nuclear matrix elements between the initial  $|J_i\rangle$  and the final  $|J_f\rangle$  states are determined by solving the BCS equations for the ground state [10,11,12] and the pn-QRPA equations for the excited states [10,11,12].

For the calculations of the original and stellar cross sections, a quenched value of  $g_A = 1.00$  is considered which subsequently modifies all relevant multipole matrix elements [2,4].

### Original Electron Capture Cross Sections

The original cross sections for the  $e^-$ -capture process in the studied isotopes are obtained by using the pn-QRPA method considering all the accessible transitions of the final nucleus. In the Donnelly-Walecka formalism the expression for the differential cross section in  $e^-$ -capture by nuclei reads [2]

$$\begin{aligned} \frac{d\sigma_{ec}}{d\Omega} = & \frac{G_F^2 \cos^2 \theta_c}{2\pi} \frac{F(Z, E_e)}{(2J_i + 1)} \left\{ \sum_{J \geq 1} \mathcal{W}(E_e, E_\nu) \right. \\ & \times \{ [1 - \alpha \cos \Phi + b \sin^2 \Phi] [|\langle J_f | \widehat{\mathcal{T}}_J^{mag} | J_i \rangle|^2 + |\langle J_f | \widehat{\mathcal{T}}_J^{el} | J_i \rangle|^2] \\ & - \left[ \frac{(\varepsilon_i + \varepsilon_f)}{q} (1 - \alpha \cos \Phi) - d \right] 2 \text{Re} \langle J_f | \widehat{\mathcal{T}}_J^{mag} | J_i \rangle \langle J_f | \widehat{\mathcal{T}}_J^{el} | J_i \rangle^* \\ & + \sum_{J \geq 0} \mathcal{W}(E_e, E_\nu) \{ (1 + \alpha \cos \Phi) |\langle J_f | \widehat{\mathcal{M}}_J | J_i \rangle|^2 \\ & + (1 + \alpha \cos \Phi - 2b \sin^2 \Phi) |\langle J_f | \widehat{\mathcal{L}}_J | J_i \rangle|^2 \\ & - \left[ \frac{\omega}{q} (1 + \alpha \cos \Phi) + d \right] 2 \text{Re} \langle J_f | \widehat{\mathcal{L}}_J | J_i \rangle \langle J_f | \widehat{\mathcal{M}}_J | J_i \rangle^* \} \end{aligned}$$

where  $F(Z, E_e)$  is the well known Fermi function. The factor  $W(E_e, E_\nu) = E_\nu^2 / (1 + E_\nu/M_T)$  accounts for the nuclear recoil [2],  $M_T$  is the mass of the target nucleus and the parameters  $a$ ,  $b$ ,  $d$  are given e.g. in Ref. [11]. The nuclear transition matrix elements between the initial state  $|J_i\rangle$  and a final state  $|J_f\rangle$  correspond to the Coulomb, longitudinal, transverse electric and transverse magnetic multipole operators.

From the energy conservation in the reaction (1), the energy of the outgoing neutrino  $E_\nu$  is written as  $E_\nu = E_e - Q + E_i - E_f$

which includes the difference between the initial  $E_i$  and the final  $E_f$  nuclear states. The  $Q$  value of the process is determined from the experimental masses of the parent ( $M_i$ ) and the daughter ( $M_f$ ) nuclei as  $Q = M_f - M_i$  [1].

Under laboratory conditions for the calculations of the original electron capture cross sections, we assumed that i) the initial state of the parent nucleus is the ground state  $|0^+\rangle$  and ii) the nuclear system is under laboratory conditions (no temperature dependence of the cross sections is needed).

The obtained total original  $e^-$ -capture cross sections for  $^{56}\text{Fe}$  and  $^{66}\text{Zn}$  target nuclei are illustrated in Fig. 1 where the individual contributions of various multipole channels are also shown.

The electron capture cross sections in Fig. 1 exhibit a sharp increase by several orders of magnitude within the first few MeV above energy-threshold, and this reflects the  $GT^+$  strength distribution. For electron energy  $E_e 10\text{MeV}$  the calculated cross sections show a moderate increase.

From experimental and astrophysical point of view, the important range of the incident electron energy  $E_e$  is up to 30 MeV. At these energies the  $1^+$  multipolarity has the largest contribution to the total electron capture cross sections [1,2]. In the present work we have extended the range of  $E_e$  up to 50 MeV since at higher energies (around 40 MeV) the contribution of other multipolarities like  $1^-$ ,  $0^+$  and  $0^-$  become noticeable and can not be omitted (see Fig. 1).

FIG. 1. Original total cross sections of electron-capture on the  $^{56}\text{Fe}$  and  $^{66}\text{Zn}$  (parent) nucleus calculated with pn-QRPA method as a function of the incident electron energy  $E_e$ . The individual contributions of various multipole channels (for  $J^\pi \leq 5^\pm$ ) are also demonstrated.

### **Stellar Electron Capture Cross Sections**

As it is well known, electron capture process plays a crucial role in late stages of evolution of a massive star, in presupernova and in supernova phases [8,13,14]. In presupernova collapse, i.e. at densities  $\rho 10^{10} \text{ gr/cm}^3$  and temperatures 300keV-800keV, electrons are captured by nuclei with  $A \approx 60$  [1,2,3,4]. During the collapse phase, at higher densities  $\rho 10^{10} \text{ gr/cm}^3$  and temperatures  $T \approx 1.0 \text{ MeV}$ , electron capture process is carried out on heavier and more neutron rich nuclei with  $Z < 40$  and  $N > 40$  [2,3,4].

For astrophysical environment, where the finite temperature and the matter density effects can not be ignored (the initial nucleus is at finite temperature), in general, the initial nuclear state needs to be a weighted sum over an appropriate energy distribution. Then, assuming Maxwell-Boltzmann distribution of the initial state  $|i\rangle$  [1,9], the total  $e^-$ -capture cross section is given by the expression [2]

$$\sigma(E_e, T) = \frac{G_F^2 \cos^2 \theta_c}{2\pi} \sum_i F(Z, E_e) \frac{(2J_i + 1) e^{-E_i/(kT)}}{G(Z, A, T)} \times \sum_{f, J} (E_e - Q + E_i - E_f)^2 \frac{|\langle i | \hat{O}_J | f \rangle|^2}{(2J_i + 1)}$$

The sum over initial states in the latter equation denotes a thermal average of levels, with the corresponding partition function  $G(Z, A, T)$  [2]. The finite temperature induces the thermal population of excited states in the parent nucleus. In the present work we assume that these excited states in the parent nucleus are all the possible states up to about 2.5 MeV. Calculations involving in addition other states lying at higher energies shows that they have no sizeable contribution to the total electron capture cross sections.

The results coming out of the study of electron capture cross sections under stellar conditions are shown in Fig. 2 where the same picture as in the original cross section calculations, but now with larger contribution is observed. As discussed before, the dominant multipolarity is the  $1^+$ , which contributes more than 40% into the total cross section. In the region of low energies (up to 30 MeV),

the total  $e^-$ -capture cross section can be described by taking into account only the GT transitions, but at higher incident energies the contributions of other multipolarities become significant and can not be omitted.

FIG. 2. Electron-capture cross sections for the  $^{56}\text{Fe}$  and  $^{66}\text{Zn}$  parent nucleus at high temperature ( $T=0.5$  MeV) in stellar environment obtained assuming Maxwell-Boltzmann statistics for the incident electrons. The total cross section and the dominant individual multipole channels ( $J^\pi \leq 5^\pm$ ) are demonstrated as functions of the incident electron energy  $E_e$ .

### **Summary and Conclusions**

The electron capture on nuclei plays crucial role during the pre-supernova phase and also in the collapse phase of stellar evolution. It becomes increasingly possible, as the density in the star's center is enhanced, and it is accompanied by an increase of the chemical potential of the degenerate electron gas. In this work, by using our numerical approach, based on a refinement of the pn-QRPA that describes reliably all the semi-leptonic weak interaction processes in nuclei, we studied in detail the electron capture process on  $^{56}\text{Fe}$  and  $^{66}\text{Zn}$  isotopes and calculated original as well as stellar  $e^-$ -capture cross sections.

Our future plans are to perform similar calculations for other interesting nuclei (like  $^{48}\text{Ti}$ ,  $^{90}\text{Zr}$  etc). Also this method could be applied to other semi-leptonic nuclear processes like beta-decay and charged-current neutrino-nucleus processes important in nuclear astrophysics and neutrino nucleosynthesis.

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