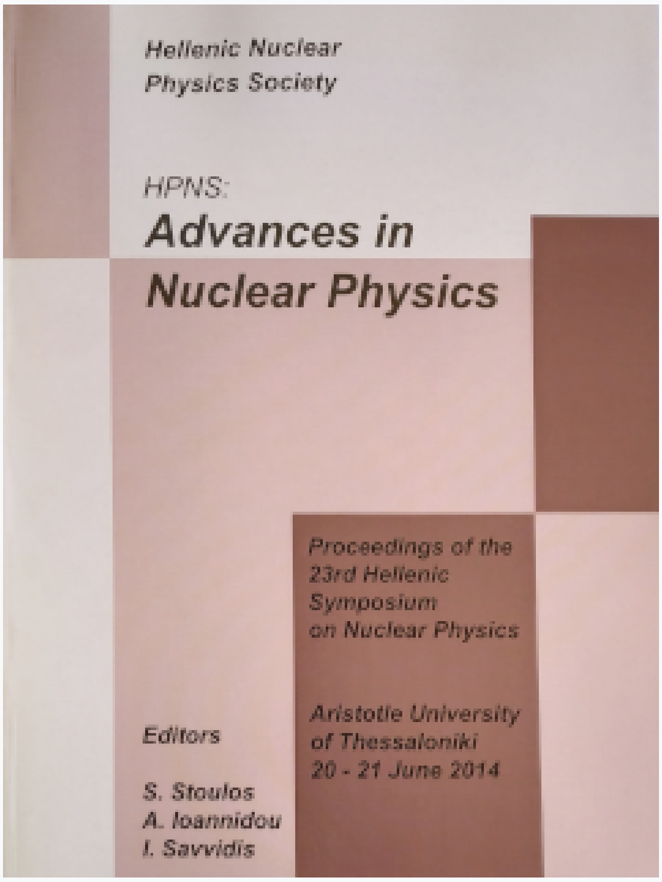
HNPS Advances in Nuclear Physics

Vol 22 (2014)

HNPS2014



Symmetry energy effects on isovector properties of neutron rich nuclei with Thomas-Fermi approach

M. C. Papazoglou, Ch. C. Moustakidis

doi: [10.12681/hnps.1933](https://doi.org/10.12681/hnps.1933)

Symmetry energy effects on isovector properties of neutron rich nuclei with Thomas-Fermi approach

M.C. Papazoglou[1], Ch. C. Moustakidis[1]

[1]*Department of Physics, Aristotle University of Thesaloniki, 54124, Thessaloniki, Greece.*

Abstract
At the present study we employ a variational method, in the framework of the Thomas-Fermi approximation, to study the effect of the symmetry energy on several isovector properties of various neutron rich nuclei. The motivation of the present work is twofold. Firstly we tried to construct a self-consistent and easily applicable density functional method to study the effect of the symmetry energy on the isovector structure properties of medium and heavy neutron rich nuclei. Secondly, our aim is, if it is possible, to combine our theoretical estimation with the relevant experimental or empirical data in order to suggest constraints on the density dependence of the symmetry energy for densities close to those of the interior of finite nuclei. According to the results, we confirm the strong dependence of the symmetry energy on the various isovector properties for the relevant nuclei, using possible constraints between the slope and the value of the symmetry energy at the saturation density.

Keywords: slope of the symmetry energy, isovector properties, variational method, self-consistent method

Introduction

The nuclear symmetry energy (SE) is the basic regulator of the isospin properties of the neutron rich nuclei [1, 2]. It is expected to affect the isovector properties of the nucleus. In addition, the density dependence of the SE is the main ingredient of the equation of state of neutron rich nuclear matter. However, the experimental data for the SE still remain limited and only for low values of density ($\rho < \rho_0$) are accurately constrained. From the theoretical point of view there is an effort to constrain the trend of SE, even for low values of density, from finite nuclei properties and to extrapolate in a way to densities related to neutron stars equation of state (up to $\simeq 5\rho_0$). In any case, the constraints of L or in general the density dependence of SE, even for low values of ff, are very important for astrophysical applications [2, 3, 4, 5].
The structure of a heavy nucleus is a result of the interplay between the strong short range nuclear forces and long range Coulomb interaction. However, in order to exhibit the isovector character of nuclear forces, we have to focus mainly on heavy and additional neutron rich nuclei. We employ a variational approach, based on the Thomas-Fermi approximation, by suitably constructing an energy density functional, and solving the derived Euler-Lagrange equation. Special attention is devoted to the contribution of the nuclear symmetry energy and the self-consistent treatment of the Coulomb interaction.
The energy density functional is a natural extension of the Bethe - Weizsacker formula, where now the total energy is a functional of the proton and neutron densities and consists of terms corresponding with those appearing in relation (1). The minimization of the total energy defines the related densities and consequently the contribution of each term separately. In the present work we apply the energy density formalism, where the total energy of finite nuclei is a functional of the total density $\rho(r)$ and the isospin asymmetry function $a(r)$, that is

$$E[\rho(r), a(r)] = \int_V \varepsilon(\rho(r), a(r)) d^3r, \qquad (1)$$

where $\varepsilon(r)$ is the local energy density. The integration is performed over the total volume V occupied by the relevant nuclei. Now we consider the functional:

$$E[\rho,\alpha]=\int_V\left[\epsilon_{ANM}\left(\rho(r),\alpha(r)\right)+F_0|\nabla\rho(r)|^2+\frac{1}{4}\rho(1-\alpha)V_c(r)\right]d^3r \quad (2)$$

The density $\rho(r)$ and the asymmetry function $\alpha(r)$ must obey the following constraints

$$\int\rho(r)d^3=A\,,\qquad \int\alpha(r)\rho(r)d^3r=N-Z\,. \quad (3)$$

The functional (2) and the constraints (3) constitute a variational problem with constraints. In the present study we consider the trial function given by the Fermi type formula

$$\rho(r)=\frac{n_0}{1+e^{(r-d)/w}}\,. \quad (4)$$

In addition, for the basic ingredients of the energy functional (2) we consider a model where the energy of the symmetric nuclear matter is given by

$$\epsilon_{SNM}(\rho)=\rho T_0\left(au^{2/3}-bu+cu^{5/3}\right),\quad u=\rho/\rho_0\,. \quad (5)$$

The total energy density of the asymmetric nuclear matter is

$$\epsilon_{ANM}(\rho,\alpha)=\rho T_0\left(au^{2/3}-bu+cu^{5/3}\right)+\alpha^2\rho J u^{\gamma}\,, \quad (6)$$

where $T_0=37.0206\ MeV$ and $\rho_0=0.16144\ fm^{-3}$ (the saturation density). The corresponding constants are: $a=-0.08203$, $b=0.97342$ and $c=0.61687$. Here we employ the simple parameterization

$$S(\rho)=S(\rho_0)\left(\frac{\rho}{\rho_0}\right)^{\gamma}=Ju^{\gamma}\,,\qquad S(\rho_0)=J\,. \quad (7)$$

In this case the parameter γ is related with both the slope L and J by the expression

$$\gamma=\frac{L}{3J}\,. \quad (8)$$

For each specific set of the Fermi type distribution parameters n_0, d, and w and a given symmetry energy $S(\rho)$, we calculate the asymmetry density $\alpha(r)$ and the total energy of the specific nucleus. After finding the density $\rho(r)$ and asymmetry function $\alpha(r)$ which minimizes the total energy, all the relevant quantities are easily calculated. In this approach α_A is deffned by the integral

$$\alpha_A=\frac{A}{(N-Z)^2}\int\rho(r)S(\rho)\alpha^2(r)d^3r\,. \quad (9)$$

One of the most important quantities concerning the isovector character of the nuclear forces is the neutron skin thickness deffned as

$$R_{skin}=R_n-R_p\,, \quad (10)$$

where R_n and R_p are the neutron and proton and radii respectively. Actually, Rskin is not directly dependent on $S(\rho)$, compared to the case of α_A, but indirectly via the dependence of $\alpha(r)$. However, recent studies conjecture that R_{skin} is a strong indicator of the isospin character of the nuclear interaction expected to be strongly correlated with the symmetry energy slope L and the value J or in general with the values of the symmetry energy close to the saturation density.

Results and Discussion

In the present work we employ a variational treatment of an energy functional, without any additional constraints requiring just the minimization of the binding energy. This approach will be suitable if we intend to impose stronger constraints on the values of L and J and might be of interest for future work.

In Fig. 1, the symmetry energy versus the total density is plotted, according to Eq. (7) for various values of the slope parameter L . It is noted that lower values of L , for low values of densities ($\rho < \rho_0$), correspond to higher values of $S(\rho)$. This behavior of $S(\rho)$ is well reflected on the values of the total binding energy E_{tot} and the asymmetry function $a(r)$.

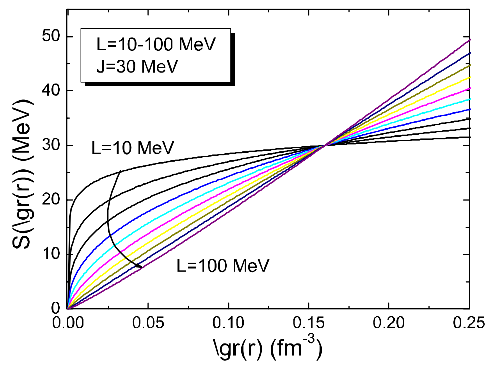


Figure 1: (Color online) The nuclear symmetry energy $S(\rho)$, deffned in Eq. (7), as a function of the density ρ for various values of the slope parameter L and the speciffc value $J = 30$ MeV.

In Fig. 2 we plot the density distributions (total, proton and neutron) as well as the corresponding asymmetry function $a(r)$ for various values of L for ^{208}Pb . The softness symmetry energy (higher values of L) shift the neutron distribution to the outer part of the nucleus, while at the same time it concentrates deeper the protons. The effects of the symmetry energy is even more pronounced on the trend of the asymmetry function.

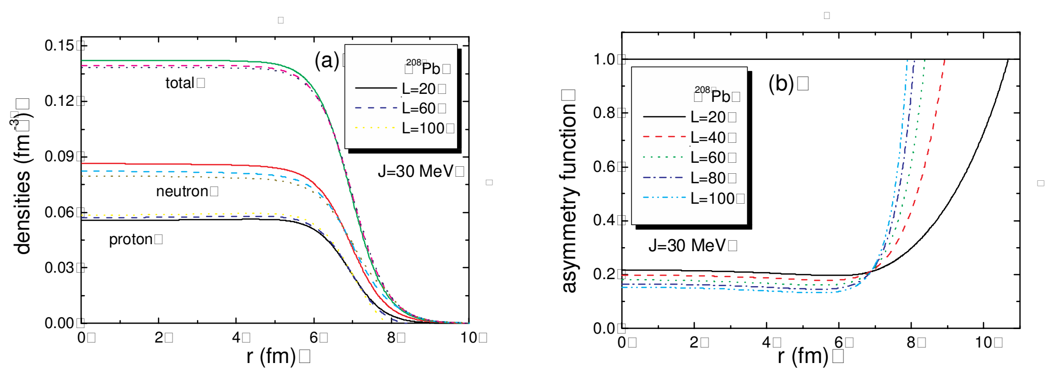


Figure 2: (Color online) The density distribution of neutrons, protons and the total one (2(a) for ^{208}Pb for three values of L (figures 2(a)) and the corresponding asymmetry functions $a(r)$ for a variety of values of L (figures 2(b)).

Fig. 3(a) displays the neutron skin R_{skin} as a function of L for various values of J for ^{208}Pb . The most striking feature is, in all cases, the strong dependence of R_{skin} on L . For a comparison, we include for the case of ^{208}Pb an approximate linear dependence

$$R_{skin}(fm) = 0.101 + 0.00147\, L\,(MeV), \tag{11}$$

established by Centelles et. al., [7] using a wide range of non-relativistic and relativistic models. It is obvious that relation (11) supports a softer dependence of R_{skin} on L compared to the present study. However we note that we present a systematic study of the effects of L on R_{skin} and in a large range of values of L without trying to reproduce for example the experimental value of the binding energy or the charge radius of the specific nucleus. The Lead Radius Experiment (PREX) at the Jefferson

Laboratory has provided the first model-independent evidence for the existence of a neutron-rich skin in ^{208}Pb [8, 9, 10, 11].

In Fig. 3(b) we display the coeffcients a_A as a function of L, for various values of J. a_A is a decreasing function of L. Actually, for speciffc pairs of values of N and Z a softer $S(\rho)$ (high values of L) leads to a lower value of $\alpha(r)$. Obviously, a_A exhibits a mass depended A behavior.

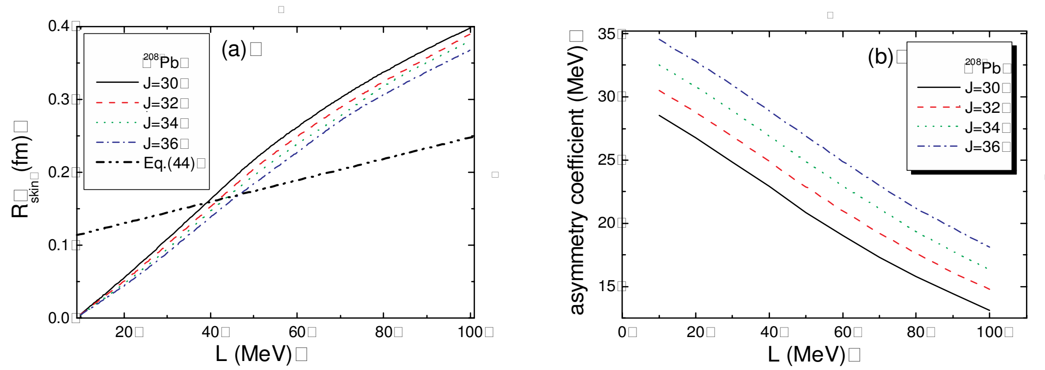


Figure 3: (Color online) (a) The neutron skin Rskin for ^{208}Pb as a function of the symmetry energy slope L, for various values of J. (b) The asymmetry coeffcient a_A for ^{208}Pb as a function of the symmetry energy slope L for various values of the parameter J.

In order to impose some possible constraints on the values of L, we plot in Fig. 4(a) for each of four nuclei ^{208}Pb, ^{124}Sn, ^{90}Zr and ^{48}Ca the pairs of L and J consistent with the corresponding empirical values of a_A. It is seen that the set $J = 34\ MeV$ and $L = 65\ MeV$ reproduces very well the empirical values of a_A for almost all the medium and heavy isotopes. Furthermore, since the four almost linear curves are arranged very close with a similar slope, we may conjecture that a possible universal dependence holds between L and J for nuclei at least in the mass region $A = 40 - 200$. For a comparison we include also the formula

$$a_A^{-1} = (a_V)^{-1} + (a_S)^{-1} A^{-1/3}, \tag{12}$$

where we use for the volume and surface coefficients $a_V = 35.5\ MeV$ and $a_S = 9.9\ MeV$ respectively. The first set ($L = 70\ MeV$ and $J = 32\ MeV$) reproduces on the average the binding energies of the corresponding isotopes while the second set ($L = 65\ MeV$ and $J = 34\ MeV$) reproduces on the average the empirical values of a_A for isotopes ^{208}Pb, ^{124}Sn, ^{90}Zr and ^{48}Ca.

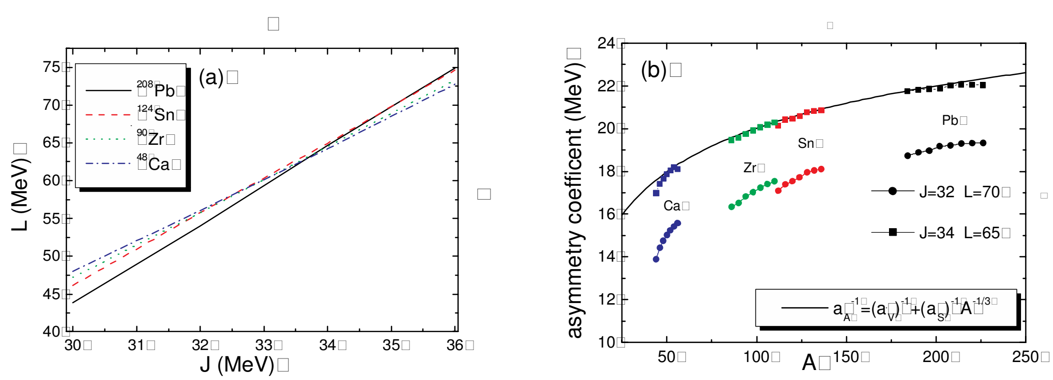


Figure 4: (Color online) (a) The plot of the pairs L and J which reproduce the empirical value of a_A given by (12) for four nuclei. (b) The asymmetry coefficients a_A as a function of A for the relevant isotopes and for the

set $L = 70$, $J = 32$ and $L = 65$, $J = 34$. The solid line corresponds to the empirical formula (12).

In Fig. 5 we compare the allowed pairs of L and J constrained from heavy-ion collisions and nuclear structure observable [6] with those found in the present approach. Actually the present results lie inside the intersection area suggested by the measurements of the dipole polarizability a_D as well as those found by heavy-ion collisions experiments.

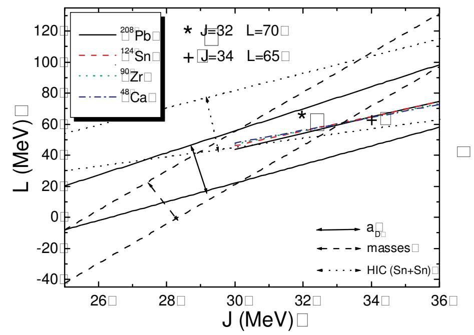


Figure 5: (Color online) Regions of allowed values of pairs J and L (three bands) constrained from heavy-ion collisions (HIC(Sn+Sn) case) and nuclear structure observables (A_D and nuclear masses) (for more details see Ref. [6]) in comparison with the corresponding results constrained from the present approach.

Conclusion

In the present work we employ a variational method, in the framework of the Thomas-Fermi approximation, in order to study the symmetry energy effects on isovector properties of various neutron rich nuclei. All the calculated properties are studied as a function of the slope of the symmetry energy and the value of the symmetry energy at the nuclear saturation density. Since, the SE even for low values of nuclear matter is uncertain, the above parameterization is necessary. We find that the neutron skin thickness is very sensitive to L i.e. it increases rapidly with L . This is expected at least in the present approximation, since the main ingredient of the relevant calculated integrals, the function $a(r)$ approaches unity very rapidly close to the critical value of r_c (at the surface of the proton distribution). Our findings, from the present study, show that the experimental knowledge of the symmetry energy at the saturation density J will impose, via the values of the symmetry coefficient a_A , strong constraints on L .

In any case, further experimental and theoretical work is necessary for a more detailed exploration of the effects of the symmetry energy on the properties of finite nuclei as well as on the neutron star structure.

References

[1] P. Danielewicz, et. al., Science 298, 1592 (2002).
[2] J.M. Lattimer and M. Prakash, Phys. Rep. 442, 109 (2007).
[3] B.A. Li, L.W. Chen, C.M. Ko, Phys. Rep. 464, 113 (2008).
[4] A.W. Steiner, M. Prakash, J.M. Lattimer, and P.J. Ellis, Phys. Rep. 411, 325 (2005).
[5] J.M. Lattimer, Annu. Rev. Nucl. Part. Sci. 62, 485 (2012).
[6] C.J. Horowitz, E.F. Brown, Y. Kim, W.G. Lynch, R. Michaels, A. Ono, J. Piekarewicz, M.B. Tsang, and H.H. Wolter, arXiv:1401.5839 [nucl-th].
[7] M. Centelles, X. Roca-Maza, X. Vi~nas, and M. Warda, Phys. Rev. Lett., 102, 122502 (2009).
[8] F.J. Fattoyev and J. Piekarewicz, Phys. Rev. C 86, 015802 (2012).
[9] F.J. Fattoyev, J. Carvajal, W.G. Newton, and B.A. Li, Phys. Rev. C 87, 015806 (2013).
[10] S. Abrahamyan, Z. Ahmed, H. Albataineh, K. Aniol, D. Armstrong et al, Phys. Rev. Lett., 108, 112502 (2012).
[11] C. Horowitz, Z. Ahmed, C. Jen, A Rakhman, P. Souder et al, Phys. Rev. C 85, 032501 (2012).
[12] W.D. Myers and W.G. Swiatecki, Phys. Rev. C 57, 3020 (1998).