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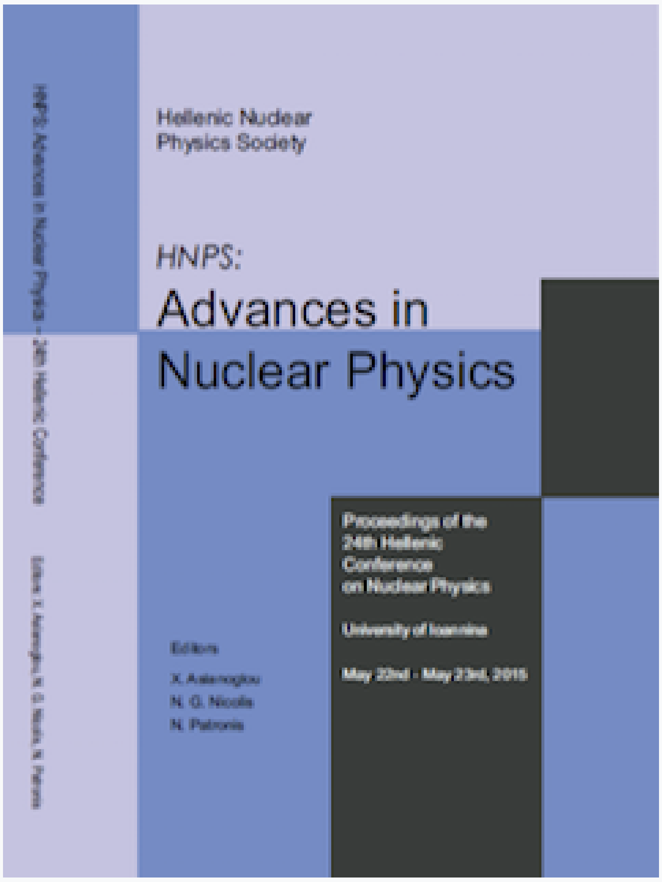

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Non-Linear Derivative Interactions in Relativistic Hadrodynamics

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Abstract

The Lagrangian of Relativistic Hadrodynamics (RHD) is extended by introducing non- linear derivative (NLD) operators into the interactions between the nucleon with the meson fields. As the novel feature of the NLD model, the nucleon selfenergy depends on both, on energy and density. Our approach contains a single cut-off parameter, which determines the density dependence of the nuclear equation of state (EoS) and the energy dependence of the nucleon-nucleus optical potential. The NLD formalism is compatible with results from microscopic nuclear matter calculations as well as with Dirac phenomenology.

Keywords: relativistic hadrodynamics, non-linear derivative model, nuclear matter, Schrödinger equivalent optical potential

1. Introduction

Relativistic mean-field (RMF) models have been widely established as a successful tool for the theoretical description of different nuclear systems such as nuclear matter, finite nuclei and heavy-ion collisions [1]. The major advantage of RMF models has been a correct description of the saturation mechanism and simultaneously an explanation of the strong spinorbit force. An energy dependence of the Schrödinger equivalent optical potential is naturally included in RMF as a consequence of a relativistic description, but it is not consistent with Dirac phenomenology [2].

We have developed a manifestly covariant model, which generates both, the correct density and, in particular, momentum dependence of the selfenergies in an unified framework. The proposed model is simple in realization and respects all the underlying symmetries of the RHD Lagrangian. The main idea was to extend the original Lagrangian of RHD [3] by including *non-linear* derivative interactions of meson fields with nucleons. In contrast to conventional RHD, the NLD Lagrangian contains all higher order derivatives of the Dirac field. Therefore the standard canonical formalism had to be generalized [4]. Although the complex structure of the canonical equations (EulerLagrange equations of motion, Noether Theorem), the NLD model gives field equations with very simple structure in nuclear matter. As an important result, both the equation of state (EoS) (density dependence) and the optical potential (energy dependence) are quantitatively well reproduced with a single parameter. Furthermore, the results are comparable with microscopic Dirac-Brueckner-Hartree-Fock (DBHF) models

1. NLD Formalism

The basis of the NLD formalism [4] builds the Lagrangian density of RHD [3]. It describes the interaction of nucleons through the exchange of virtual meson fields (Lorentz- scalar, σ, and Lorentz-vector meson fields ω^{μ})

$$\mathcal{L} = \mathcal{L}_{Dirac} + \mathcal{L}_{mes} + \mathcal{L}_{int} \; . \tag{1}$$

The Lagrangian in Eq. (1) consists of the free Lagrangians for the nucleon field Ψ and for the meson fields σ and ω^{μ}. In standard RHD the interaction Lagrangian L_{int} contains meson fields which couple to the Dirac spinors via the Lorentz density operators $\Psi\Psi\sigma$ and $\Psi\gamma^{\mu}\Psi\omega_{\mu}$ with given coupling constants g_{σ} and g_{ω}, respectively. Such interactions give rather successful saturation properties of nuclear matter, but they don't describe the energy dependence of the mean-field correctly. For this reason we have generalized the standard RHD by introducing nonlinear derivative operators into the interaction Lagrangian density

$$\mathcal{L}_{int} = \frac{g_\sigma}{2}\left[\overline{\Psi}\overleftarrow{\mathcal{D}}\Psi\sigma + \sigma\overline{\Psi}\overrightarrow{\mathcal{D}}\Psi\right] - \frac{g_\omega}{2}\left[\overline{\Psi}\overleftarrow{\mathcal{D}}\gamma^\mu\Psi\omega_\mu + \omega_\mu\overline{\Psi}\gamma^\mu\overrightarrow{\mathcal{D}}\Psi\right] \; . \tag{2}$$

The interaction between the spinor fields Ψ, Ψ and the meson fields has a similar functional form as in standard RHD [3]. However, now new operators D acting on the nucleon fields
appear

$$\overrightarrow{\mathcal{D}} := \exp\left(\frac{-v^\beta i \overrightarrow{\partial}_\beta + m}{\Lambda}\right) , \quad \overleftarrow{\mathcal{D}} := \exp\left(\frac{i\overleftarrow{\partial}_\beta v^\beta + m}{\Lambda}\right) . \tag{3}$$

In Eq. (3) v^{μ} is a dimensionless auxiliary 4-vector. Λ is a cutoff parameter which has been adjusted to the saturation properties of nuclear matter, and m is the nucleon mass. In the limiting case of $\Lambda \rightarrow \infty$ the standard RMF or Walecka model is retained.

The NLD Lagrangian L is a functional of not only Ψ, $\overline{\Psi}$ and their first derivatives, but it depends on all higher order covariant derivatives of the spinor fields Ψ and $\overline{\Psi}$. For such a generalized functional the Euler-Lagrange equations take the form [4]

$$\frac{\partial\mathcal{L}}{\partial\varphi} - \partial_\alpha\frac{\partial\mathcal{L}}{\partial(\partial_\alpha\varphi)} + \partial_\alpha\partial_\beta\frac{\partial\mathcal{L}}{\partial(\partial_\alpha\partial_\beta\varphi)} + \cdots + (-)^n\partial_{\alpha_1}\partial_{\alpha_2}\cdots\partial_{\alpha_n}\frac{\partial\mathcal{L}}{\partial(\partial_{\alpha_1}\partial_{\alpha_2}\cdots\partial_{\alpha_n}\varphi)} = 0 \; , \tag{4}$$

where ϕ stands for the NLD degrees of freedom (Dirac Spinor Ψ and meson fields σ and ω).

Contrary to the standard expressions for the Euler-Lagrange equation, now infinite series of terms proportional to higher order derivatives of the Dirac field appear. They can be evaluated by a Taylor expansion of the non-linear derivative operators (3). As shown in [4], all infinite series can be resumed and all the equations simplify considerably. Applying the usual RMF approximation to infinite nuclear matter, the standard Dirac equation is obtained

$$[\gamma_\mu(i\partial^\mu - \Sigma^\mu) - (m - \Sigma_s)]\,\Psi = 0\;, \tag{5}$$

with selfenergies given by

$$\Sigma_v = g_\omega \omega_0 e^{-\frac{E-m}{\Lambda}}\;,\; \Sigma_s = g_\sigma \sigma e^{-\frac{E-m}{\Lambda}}\;. \tag{6}$$

The relation between the particle momentum p and the particle energy E is obtained from the in-medium mass-shell condition

$$E(p) = \sqrt{p^2 + m^{*2}} + \Sigma_v \tag{7}$$

with the in-medium (or effective) Dirac mass given by $m^* = m - \Sigma_S$. Note, that the selfenergies and thus the effective mass m^* explicitly depend on particle momentum. For the limiting case when $\Lambda \to \infty$, the exponential factors are equal to unity and the equations are reduced to the ones from the Walecka model. The NLD selfenergies contain a non-linear energy dependence, as also expected from Dirac Phenomenology [2]. In nuclear matter the NLD equations of motion for ω and σ simplify to standard meson field equations

$$m_\omega^2 \omega^0 = g_\omega \rho_0\;,\; m_\sigma^2 \sigma = g_\sigma \rho_s\;. \tag{8}$$

With the corresponding density sources $\rho_s = \langle \overline{\Psi} e^{-\frac{E-m}{\Lambda}} \Psi \rangle$ and $\rho_0 = \langle \overline{\Psi} \gamma^0 e^{-\frac{E-m}{\Lambda}} \Psi \rangle$. Note that the density ρ_0 is not related to the conserved nucleon density ρ_B. It has to be derived from a generalized Noether-theorem [4] and reads

$$J^0 \equiv \rho_B = \langle \overline{\Psi} \gamma^0 \Psi \rangle + \frac{g_\omega}{\Lambda} \langle \overline{\Psi} \gamma^0 e^{-\frac{E-m}{\Lambda}} \Psi \rangle \omega_0 - \frac{g_\sigma}{\Lambda} \langle \overline{\Psi} e^{-\frac{E-m}{\Lambda}} \Psi \rangle \sigma \quad . \tag{9}$$

The NLD model contains no free parameters except Λ, since in the limiting cases the conventional Walecka model is retained. The original meson-nucleon couplings can be taken from any linear Walecka model, e.g., [3], as it has been done here. The cut-off parameter Λ is of natural size, i.e., of typical hadronic mass scale in this problem. In the following, Λ = 0.770 GeV is chosen [4].

2. Results and Discussion

We have applied both the NLD approach and the linear Walecka model to infinite nuclear matter at various baryon densities and nucleon energies relative to nuclear matter at rest.

According Eqs. (6) the cut-off parameter Λ generates a highly non-linear density dependence for nuclear matter at rest, which arises from the exponential terms. This non-linear density dependence affects considerably the equation of state (EoS), i.e., the binding energy per nucleon as function of nucleon density. This is demonstrated in Fig. 1 for nuclear matter and also pure neutron matter (for neutron matter the isovector ρ-meson was included). First of all, the conventional linear Walecka model (dashed curve) leads to an EoS with high stiffness. This results to a very high value for the compression modulus. The NLD model weakens the stiffness of the EoS for nuclear and pure neutron matter to a large extent. The agreement of the NLD-EoS with the underlying DB theory is successful indicating that saturation nuclear matter properties, e.g., binding energy per nucleon

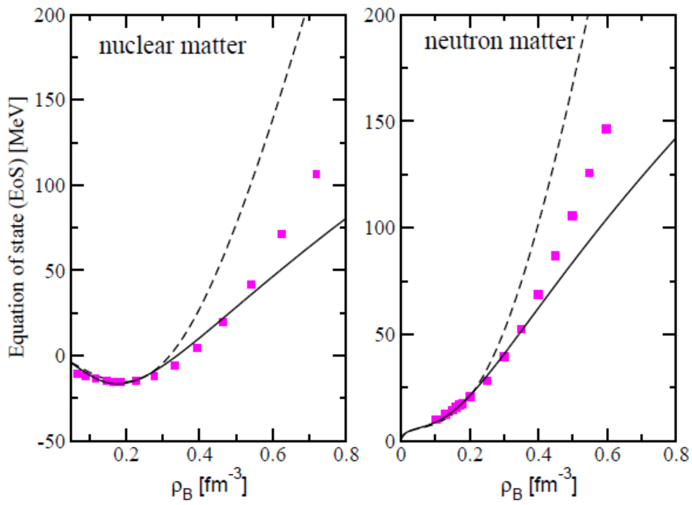

Figure 1: Equation of state for nuclear (left panel) and pure neutron matter (right panel). Dashed: linear Walecka model, solid: NLD model, filled squares: DBHF model [5].

and compression modulus at saturation density, are fairly well reproduced by the NLD approach. Similar effects are observed for the density dependence of the effective mass (not shown here). At saturation density a value of $m^* \approx 0.65m$ is obtained, which is again

very close to DBHF predictions. Therefore, it turns out that conventional RMF models without any explicit introduction of selfinteractions of the meson fields are able to describe nuclear matter properties, if non-linear derivatives are accounted for in the underlying meson-nucleon interaction Lagrangian.
A novel feature of the NLD model is the prediction of an energy (or momentum) dependence of the nuclear mean-field with only one parameter. Dirac Phenomenology on elastic proton-nucleus scattering predicts a non-linear energy dependence of the Schrödinger equivalent optical potential, which cannot be reproduced in standard linear Walecka models nor in their extensions to non-linear meson field terms. The question arises if the NLD model can reproduce this feature with the same parameter Λ as used for the density dependence. For this purpose, we consider the situation of a nucleon with particular momentum *p* (or kinetic energy E_{kin}) relative to nuclear matter at rest. The kinetic energy for an incident
free nucleon with mass *m* and momentum *p* is usually defined as

$$E_{kin} = \sqrt{p^2 + m^2} - m$$

In the nuclear medium one has to determine the kinetic energy relative to the potential
deep [5]

$$E_{kin} = E - m = \sqrt{p^2 + m^{*2}} + \Sigma_v - m \quad . \tag{10}$$

The energy dependence of the nuclear mean-field is empirically determined by Dirac phenomenology in elastic nucleon-nucleus scattering [2]. The key quantity in empirical studies is the Schrödinger equivalent optical potential U_{opt}, which serves as a convenient

means to describe the in-medium interaction of a nucleon with momentum *p* relative to
nuclear matter at rest. It is obtained by a non-relativistic reduction of the Dirac equation and reads

$$U_{\text{opt}} = \frac{E}{m}\Sigma_v - \Sigma_s + \frac{1}{2m}\left(\Sigma_s^2 - \Sigma_v^2\right) \ . \tag{11}$$

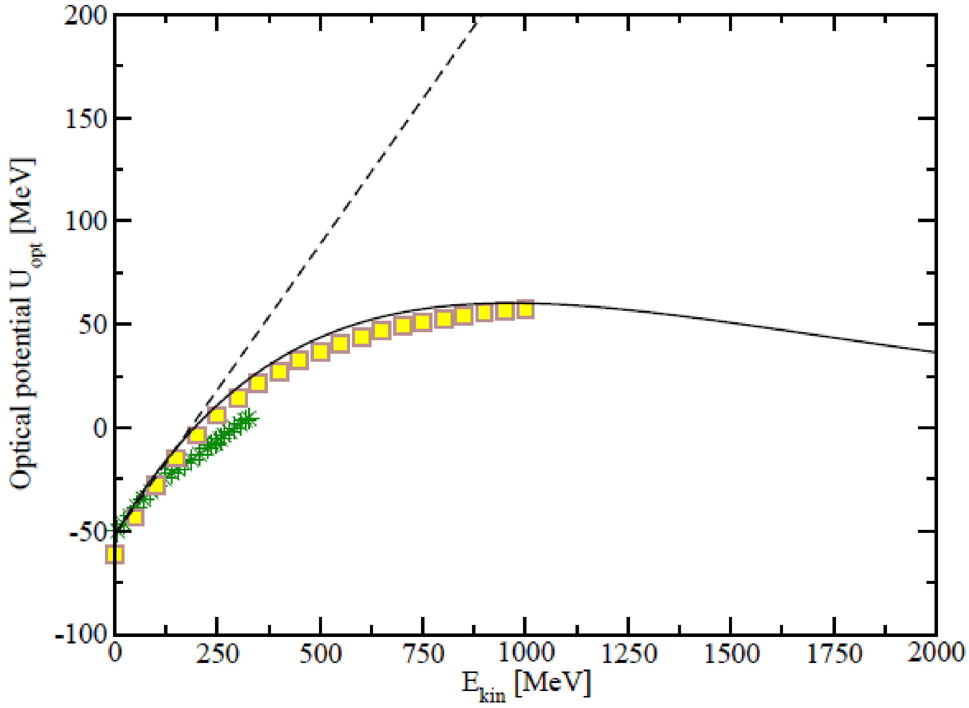

Figure 2: Energy dependence of the Schr¨odinger equivalent proton-nucleus optical potential at saturation density $\rho sat = 0.16\ fm^{-3}$. Theoretical calculations in the linear Valecka

It is fully determined by the Lorentz-scalar and Lorentz-vector components of the nucleon selfenergy. The optical potential rises linearly with energy, if the selfenergies do not depend explicitely on momentum. This is the case of the linear Walecka model, as can be seen in Fig. 2 (dashed curve). The DBHF model (filled stars), on the other hand, reproduces the empirical behavior of the optical potential only at low energies, since the parameters of the underlying free NN-interaction are fitted to low energy scattering data [5]. The NLD model (solid curve) with its non-linear energy dependence weakens strongly the linear stiffness of the original Walecka model, and the empirical energy dependence of the optical potential can be reproduced fairly well without the introduction of any further parameters.

3. Summary and Outlook

In summary, we presented the NLD formalism, which constitutes a generalization of standard RHD by imposing on a mean-field level highly non-linear effects in baryon density and simultaneously in single-particle energy. In contrast to conventional RMF models, the NLD approach contains a single cut-off parameter of natural hadronic scale, which drives the density *and simultaneously* the energy dependence of the mean-field.

We applied the NLD approach to nuclear matter and nucleon scattering with nuclear matter at rest. Astonishing were the non-linear density dependence of the vector field without the introduction of any additional selfinteraction terms in the original Lagrangian of the linear Walecka model. These non-linear effects lead to a softening of the equation of state for nuclear and pure neutron matter at high densities. It was possible to describe quantitatively well the empirically known saturation properties. The results were also comparable to predictions of microscopic DBHF calculations over a wide density range. The NLD approach lead furthermore to a momentum dependence of the selfenergies. As a novel feature of NLD, the energy dependence of the Schrödinger equivalent optical potential was reproduced fairly well by utilizing the same parameter.

The application of the NLD formalism to heavy-ion collisions in the spirit of a covariant transport theory based on the present Lagrangian would be a great challenge for the future in studying hadronic matter under extreme conditions with the ultimative goal of exploring the equation of state at supra-normal densities, as they are planned at the new FAIR facility at GSI.

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