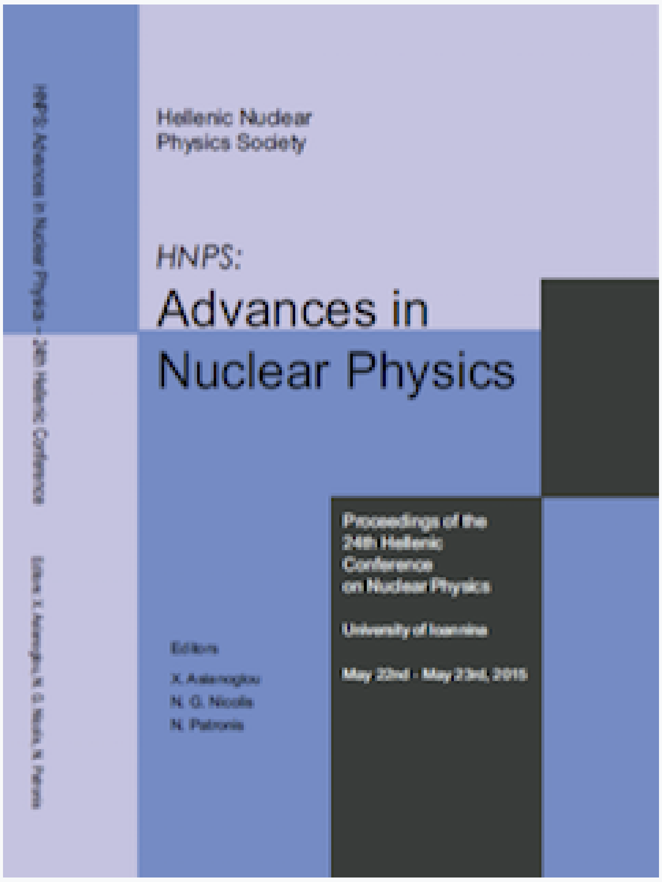
HNPS Advances in Nuclear Physics

Vol 23 (2015)

HNPS2015



R-mode constraints from neutron star equation of state

Ch. C. Moustakidis, M. C. Papazoglou

doi: 10.12681/hnps.1913

R-mode constraints from neutron star equation of state

Ch.C. Moustakidis and M.C. Papazoglou

Department of Theoretical Physics, Aristotle University of Thessaloniki, 54124 Thessaloniki, Greece

Abstract

The gravitational radiation has been proposed a long time before, as an explana- tion for the observed relatively low spin frequencies of young neutron stars and of accreting neutron stars in low-mass X-ray binaries as well. In the present work we studied the effects of the neutron star equation of state on the r-mode instability window of rotating neutron stars [1].

There are several open problems in physics and astrophysics on neutron stars [2]. One of the problems is why neutron stars do not spin up to the theoretically allowed limit called Kepler frequency. In particular, there is a sharp cut off for spins above 730, Hz which are well below the theoretically allowed upper limit [3]. One possibility is the radiation of gravitational waves from the rapidly rotating pulsars. In particular, neutron stars may suffer a number of instabili- ties which come in different flavors but they have a general feature in common; they can be directly associated with unstable modes of oscillation [4,5]. The r-modes are oscillations of rotating stars whose restoring force is the Coriolis force. The gravitational radiation-driven instability of these modes has been proposed as an explanation for the observed relatively low spin frequencies of young neutron stars and of accreting neutron stars in low-mass X-ray binaries as well [4].

The motivation of the present work is twofold. First, we intend to examine possible constraints on the r-mode instability related to the bulk neutron stars properties (mass, radius, density distribution, crust elasticity, e.t.c.) by em- ploying a suitable set of analytical solutions of TOV equations. Second, our aim is to examine and if possible to establish, relations between the critical angular velocity Ω_C and a) the nuclear equation of state via the slope param- eter L and b) the crust elasticity via the slippage factor S. In particular, we propose a correlation between Ω_C and the derivative of the nuclear symmetry energy with respect to the baryon density.

The r-modes evolve with time dependence $e^{i\omega t - t/\tau}$ as a consequence of ordinary hydrodynamics and the influence of the various dissipative processes. The real part of the frequency of these modes, ω, is given by

$$\omega = -\frac{(l-1)(l+2)}{l+1}\Omega, \quad (1)$$

where Ω is the angular velocity of the unperturbed star [6]. The imaginary part $1/\tau$ is determined by the effects of gravitational radiation, viscosity, etc. [4,7,6]. In the small-amplitude limit, a mode is a driven, damped harmonic oscillator with an exponential damping time scale

$$\frac{1}{\tau(\Omega,T)}=\frac{1}{\tau_{GR}(\Omega)}+\frac{1}{\tau_{EL}(\Omega,T)}+\frac{1}{\tau_{BV}(\Omega,T)}+\frac{1}{\tau_{SV}(\Omega,T)}+\frac{1}{\tau_{MF}(\Omega,T)},$$

where $\tau_{GR}, \tau_{EL}, \tau_{BV}$ *and* τ_{MF} are the gravitational radiation time scale, the damping time scale due to viscous dissipation at the boundary layer of the rigid crust and fluid core, the bulk and shear viscosity dissipation times scales respectively and the damping time scale due to the mutual friction. Gravitational radiation tends to drive the *r*-modes unstable, while viscosity and mutual friction suppress the instability. More precisely dissipative effects cause the mode to decay exponentially as $e^{-t/\tau}$ (i.e., the mode is stable) as long as $\tau > 0$ [6]. The damping time τ_i for the individual mechanisms is defined in general by [4]

$$\frac{1}{\tau_i}\equiv-\frac{1}{2E}\left(\frac{dE}{dt}\right)_i. \tag{2}$$

In Eq. (2) the total energy E of the r-mode is given by [4,6]

$$E=\frac{1}{2}\alpha^2R^{-2l+2}\Omega^2\int_0^R\rho(r)r^{2l+2}dr, \tag{3}$$

where α is the dimensionless amplitude of the mode, R is the radius, Ω is the angular velocity and $\rho(r)$ is the radial dependence of the mass density of the neutron star.

Firstly, we study the case where the viscosity due to boundary layer of the rigid crust is not taken into account the equilibrium equation (minimal model). Then, the equilibrium equation, $\frac{1}{\tau}=0$, is written [8]

$$-\left(\frac{\Omega_c}{\mathrm{Hz}}\right)^6+a\left(\frac{\Omega_c}{\mathrm{Hz}}\right)^2+b=0. \tag{4}$$

Eq. (4) is directly converted to a cubic equation. The above equation, in any case, can be solved numerically to give the desired critical frequency Ω_C. However, in this case, it is conceptually difficult to intuit answers. Eq. (4) can be also solved analytically and the solution is given, for $Y \leq 1$, by

$$\Omega_c=\left(\frac{b}{2}\right)^{1/6}\sqrt{\left(1+\sqrt{1-Y}\right)^{1/3}+\left(1-\sqrt{1-Y}\right)^{1/3}} \tag{5}$$

and for $Y \geq 1$ by

$$\Omega_c=\left(4b\sqrt{Y}\right)^{1/6}\sqrt{\cos\left[\frac{1}{3}\tan^{-1}\left(\sqrt{Y-1}\right)\right]} \tag{6}$$

where $Y = \dfrac{4a^3}{27b^2}$ and also

$$a = 3.237 \cdot 10^4 \left(\frac{10\mathrm{km}}{R}\right)^3 \left(\frac{M}{M_\odot}\right)^2 \left(\frac{T}{10^9\mathrm{K}}\right)^6 \frac{I_2}{I_1^2}, \tag{7}$$

$$b = 2.265 \cdot 10^{15} \left(\frac{10^9\mathrm{K}}{T}\right)^2 \left(\frac{10\mathrm{km}}{R}\right)^9 \frac{1}{I_1^2} \left[\left(\frac{\rho_c}{10^{16}\mathrm{gr\ cm^{-3}}}\right)^{1/4} I_3^{nn} + 1.729 I_3^{ee}\right]. \tag{8}$$

We also consider the effect on r-mode instability due to the presence of a solid crust in an old neutron star (minimal model+crust effects). It is proved that the presence of a viscous boundary layer under the solid crust of a neutron star increases the viscous damping rate of the fluid r-modes [4,9]. Actually, the presence of a solid crust has a crucial effect on the r-mode motion and following the discussion of [10] this effect can be understood as follows: based on the perfect fluid mode-calculations it is anticipated the transverse motion associated with the mode at the crust-core boundary to be large. However, if the crust is assumed to be rigid, the fluid motion must essentially fall off to zero at the base of the crust in order to satisfy a non-slip condition (in the rotating frame of reference).

The equilibrium equation, when the dissipation mechanism due to the crust has been included, is given now by

$$-\left(\frac{\Omega_c}{\mathrm{Hz}}\right)^6 + \tilde{a}\left(\frac{\Omega_c}{\mathrm{Hz}}\right)^2 + \tilde{d}\left(\frac{\Omega_c}{\mathrm{Hz}}\right)^{1/2} + \tilde{b} = 0, \tag{9}$$

where the coefficients $\tilde{a}$ and $\tilde{b}$ are similar with a and b, given by Eqs (7) and (8), where now the structure integrals I_i ($i = 1, 2, 3$) have been replaced by the corresponding $\tilde{I}_i$. In the present work we also explore the case of an elastic crust. In this case the r-mode penetrates the crust and consequently the relative motion (slippage) between the crust and the core is strongly reduced compared to the rigid crust limit [11]. In particular, the way the slippage factor S defined as $S = \Delta v/v$ has been included on the r-mode problem which has been discussed in Refs. [11–13]. They propose that the factor S must be included quadratically in the r-mode damping formula. This leads to a revised Ekman layer time scale [13]

$$\tau_{EL}^{S} \rightarrow \frac{\tau_{EL}}{S^2}. \tag{10}$$

Actually, the factor S depends mainly on the angular velocity Ω, the core radius R_C and the shear modulus μ but can be treated also, in approximated way, as a constant (see also [13]). In particular, in Eq. (10) the factor S is used as a free parameter varied in the interval of very low values (S = 0.05) up to the value S = 1 which corresponds to a complete rigid crust.

Finally, in the present study we also consider an additional damping mecha- nism called mutual friction (for more details see [14]). This mechanism arises from the scattering of electrons of the magnetic fields which entrapped in the cores of the superfluid neutron vortices ([14]). Mutual friction is considered as a candidate to provide the needed stability for the r-modes in old cold neutron stars while it has been shown that suppresses the gravitational radiation in the case of the f-modes of rotating neutron star. The dissipation time scale due to the mutual friction is given also by

$$\frac{1}{\tau_{MF}} = 3.2 \cdot 10^{-28} \frac{1}{\tilde{\tau}_{MF}} \left(\frac{R}{\mathrm{km}}\right)^{15/2} \left(\frac{M_{\odot}}{M}\right)^{5/2} \left(\frac{\Omega}{\mathrm{Hz}}\right)^{5}. \tag{11}$$

The characteristic damping time scale $\tilde{\tau}_{MF}$ is independent of angular velocity and temperature (to lowest order) but sensitively depends on the entrainment parameter ϵ [14]. Actually, $\tilde{\tau}_{MF}$ has typical values 10 sec, however, a resonance phenomenon leads to very small values for a few narrow range of ϵ([14]). In the present study we treat $\tilde{\tau}_{MF}$ as a phenomenological parameter varying in the range 5 s $\leq \tilde{\tau}_{MF} \leq$ 10 s according to the previous study of [14].

Now, the equilibrium equation is given by

$$\frac{\tilde{c}}{\tilde{\tau}_{MF}} \left(\frac{\Omega_c}{\mathrm{Hz}}\right)^{5} + \tilde{a} \left(\frac{\Omega_c}{\mathrm{Hz}}\right)^{2} + \tilde{d} \left(\frac{\Omega_c}{\mathrm{Hz}}\right)^{1/2} + \tilde{b} = \left(\frac{\Omega_c}{\mathrm{Hz}}\right)^{6}, \tag{12}$$

Where the coefficients $\tilde{a}, \tilde{b}$ and $\tilde{d}$ are similar with those in eq. (9) while the coefficient $\tilde{c}$ is given by

$$\tilde{c} = 4.178 \cdot 10^{17} \left(\frac{R}{\mathrm{km}}\right)^{1/2} \left(\frac{M_{\odot}}{M}\right)^{5/2} \left(\frac{\mathrm{gr\ cm}^{-3}}{\rho_c}\right) \frac{1}{\tilde{I}_1}. \tag{13}$$

Motivated by the strong radius dependence of the critical angular velocity Ω_C, we propose a phenomenological approach to study the EOS effects on the r-mode instability window. This approach, despite its simplicity, provides a few insights of the mentioned study, in a universal way, and also leads to some simplified empirical relations. Moreover, the proposed method suggests and provides, in a way, constraints on the nuclear equation of state with the help of accurate measurements of the main bulk neutron star properties.

We consider that the energy per particle of nuclear matter close to saturation density n_S, in the parabolic approximation, has the form [15]

$$E(n,x) \simeq E(n,x=\frac{1}{2}) + E_{sym}(n)(1-2x)^2. \tag{14}$$

In Eq. (14) n is the baryons density, $E_{sym}(n)$ is the symmetry energy and x is the proton fraction. $E(n,x=\frac{1}{2})$ is the energy per particle of symmetric nuclear matter, where close to the saturation density can be written in a good approximation

$$E(n,x=\frac{1}{2}) \simeq -16 + \frac{K}{18}\left(1-\frac{n}{n_s}\right)^2 + \frac{L}{162}\left(1-\frac{n}{n_s}\right)^3. \tag{15}$$

The incompressibility K and the skewness L are defined as

$$K = 9n_s^2 \frac{\partial^2 E(n,x)}{\partial n^2}\,|_{n=n_s}, \qquad L = -27n_s^3 \frac{\partial^3 E(n,x)}{\partial n^3}\,|_{n=n_s}. \tag{16}$$

In neutron star matter, in order to satisfied the β-equilibrium, a small electron fraction exists and contributes to the total energy according to the expression

$$E_e = \frac{3\hbar c}{4}(3\pi^2 n x^4)^{1/3}. \tag{17}$$

The total energy is given now by

$$E(n,x) = E(n,x) + E_e(n,x), \tag{18}$$

while the total pressure is defined as

$$P(n,x) = n^2 \frac{\partial E}{\partial n}. \tag{19}$$

The proton fraction x in β-equilibrium is regulated by the value of the symmetry energy. In particular, is determined by solving the equation $\partial E/\partial x = 0$ which leads to [16]

$$4E_{sym}(n)(1-2x) = \hbar c(3\pi^2 n x)^{1/3}. \tag{20}$$

The combination of Eqs. (18) and (19) leads to

$$\begin{aligned} P(n,x) = n^2 \Bigg[&\frac{\partial E_{sym}(n)}{\partial n}(1-2x)^2 + \frac{x E_{sym}}{n}(1-2x) \\ &- \frac{K}{9n_s}\left(1-\frac{n}{n_s}\right) - \frac{L}{54n_s}\left(1-\frac{n}{n_s}\right)^2 \Bigg]. \end{aligned} \tag{21}$$

The expression (21) has been extensively used in the literature for neutron star structure studies. In particular, the pressure at the saturation density n_S takes the form

$$P(n_s, x_s) = n_s^2 \left[\left(\frac{\partial E_{sym}(n)}{\partial n} \right)_{n_s} (1-2x_s)^2 + \frac{x_s E_{sym}(n_s)}{n_s} (1-2x_s) \right]. \quad (22)$$

Now, if we define the value of the symmetry energy at the saturation as $J = E_{sym}(n_s)$ and the slope parameter as $L = 3n_s \left(\frac{\partial E_{sym}(n)}{\partial n}\right)$, Eq. 22 is written as

$$P(n_s, x_s) = n_s \left[\frac{L}{3}(1-2x_s)^2 + x_s J (1-2x_s) \right]. \quad (23)$$

According to Eq. (23) the total pressure P at the saturation density depends directly on the slope parameter L (mainly) and J and indirectly on the men- tioned parameters via the proton fraction x_S. Since the proton fraction, for densities close to n_s is $x \ll 1$, then in a good approximation Eq. (23) takes the form

$$P(n_s, x_s) \simeq n_s \frac{L}{3}. \quad (24)$$

The expression (24) has a clear meaning, the pressure of neutron star matter close to the saturation density is directly related to the symmetry energy via the slope parameter L. The above finding became very important when Lattimer and Prakash, found a remarkable empirical relation which exists between the radii of 1 and 1.4 $M_\odot$ neutron stars and the corresponding neutron stars matter's pressure evaluated at densities 1, 1.5 and 2 of the saturation density n_S [17]. The mentioned relation obeys a power-low relation:

$$R(M) = C(n, M) \left[\frac{P(n)}{\text{MeV fm}^{-3}} \right]^{1/4}, \quad (25)$$

where $R(M)$ is the radius of a star mass M, $P(n)$ is the pressure of neutron star matter at density n and $C(n, M)$ is a number that depends on the density n at which the pressure was evaluated and the stellar mass M. The values of $C(M, n)$ for the various cases are presented in Table. 3 of Ref. [17]. These values were estimated by averaging results of 31 disparate equations of state. Recently, Lattimer and Lim [18] excluding those equations of state, because of the maximum mass constraints imposed by PSR J1614-2230 ([19]) and they found the revised value

$$C(n_s, 1.4\widetilde{M}_\odot) = 9.52 \pm 0.49 \text{ km}. \quad (26)$$

The correlation (25) is significant since the pressure of neutron star matter near the saturation density is, in large part, determined by the symmetry energy of the EOS [17]. Moreover, it relates the macroscopic quantity R (and of course all the relative quantities for example moment of inertia etc.) to the microscopic quantity P. Consequently, this formula, supports the statement that the nuclear equation of state plays an important role on the construction of relativistic very dense objects i.t. a neutron star. Moreover the formula (25), since it directly relates the radius to the slope parameter L, exhibits the dependence of the neutron star size on the nuclear symmetry and consequently on the isovector character of the nucleon-nucleon interaction. More precisely, inverting Eq. (25) yields

$$P(n) \simeq \left[\frac{R}{C(n,M)}\right]^4 \left(\mathrm{MeV\ fm}^{-3}\right), \tag{27}$$

where apparently, various restrictions on the equation of state are possible if the radius of a neutron star can be measured with high accuracy [17]. As we show the r-mode instability window, defined by the dependence Ω_C T _, is strongly affected by the neutron star radius R. The effects of the mass M and the mass distribution $\rho(r)$ play minor role. Consequently, the dominant effect of the equation of state on the r-mode is originated from the predicted values of the neutron star size. In view of the above statement, we employ the correlation (25) in order to relate the angular velocity Ω_C with effects of the

EOS and mainly the slope parameter L which consists a basic characteristic of the EOS and is related to the derivative of the symmetry energy at the saturation density.

For a static spherical symmetric system, the metric can be written as follows [2]

$$\frac{8\pi G}{c^2}\rho(r) = \frac{1}{r^2}\left(1 - e^{-\lambda(r)}\right) + e^{-\lambda(r)}\frac{\lambda'(r)}{r}, \tag{29}$$

$$\frac{8\pi G}{c^4}P(r) = -\frac{1}{r^2}\left(1 - e^{-\lambda(r)}\right) + e^{-\lambda(r)}\frac{\nu'(r)}{r}, \tag{30}$$

where derivatives with respect to the radius are denoted by ‵. The combination of Eqs (29) and (30) leads to the well known Tolman-Oppenheimer-Volkovf equations [2]

$$\frac{dP(r)}{dr} = -\frac{G\rho(r)M(r)}{r^2}\left(1 + \frac{P(r)}{\rho(r)c^2}\right)\left(1 + \frac{4\pi P(r)r^3}{M(r)c^2}\right)\left(1 - \frac{2GM(r)}{c^2 r}\right)^{-1},$$

$$\frac{dM(r)}{dr} = 4\pi r^2\rho(r). \tag{31}$$

It is difficult to obtain exact solution of TOV equations in closed analytical form and they solved numerically with an equation of state specified. Actually, there are hundreds of analytical solutions of TOV equations but three of them satisfy the criteria that the pressure and energy density vanish on the surface of the star. Also both of them decrease monotonically with increasing radius. These three solutions are the Tolman VII, the Buchdahl's and the Nariai IV. It is worth pointing out that all the analytical solutions presented and used in the present work contain two parameters, the central density ρ_C and the compactness parameter $\beta = GM/Rc^2$.

We have studied the effect of the elasticity of the crust, via the slippage factor S, on the instability window. The value $S = 1$ corresponds to a complete rigid crust without elasticity while lower values of S introduce elastic properties to the crust. [11] showed that the slippage factor is $S \approx 0.05 - 0.1$ in a typical case, while [13] found the value $S \approx 0.05$. The critical frequency Ω_C is written

$$\Omega_c \simeq 4360\ S^{4/11} \left(\frac{10^9 \mathrm{K}}{T}\right)^{2/11} \left(\frac{10 \mathrm{Km}}{R}\right)^{4/11} \quad (\mathrm{Hz}). \qquad (32)$$

Finally, we studied the effects of the mutual friction on the instability win- dow in comparison to the minimal model and the crust viscosity effects. The corresponding time scale $\tilde{\tau}_{MF}$ varying in the large range 5 s $\leq \tilde{\tau}_{MF} \leq$ 10 s in order to systematically study the mutual friction effects (see Fig. 2). We confirm the results of the previous work of [14] where the MF effects are almost negligible for $\tilde{\tau}_{MF} >$ 50 s. In this case, the main viscosity mechanism is due to the Ekman layer viscosity and the previous analysis concerning the r-mode from the equation of state is a good approximation. However, for $\tilde{\tau}_{MF} <$ 50 s the mutual friction effects are very important narrowing remarkably the in- stability window. In particular, for $\tilde{\tau}_{MF} \cong 5$ s the window disappears that is the mutual friction suppresses completely the gravitational radiation. In this case, since the mutual friction suppression overcomes significantly those due to the Ekman layer the value of the time scale $\tilde{\tau}_{MF}$ is the dominate factor and further analysis is essential in order to clarify further the role of the equation of state. Actually, in this case and in a good approximation, the equilibrium equation takes the simple form $t_{MF} = |t_{GR}|$ and the critical angular velocity Ω_C, for the Tolman VII solution, is given by

$$\Omega_c = 43.2 \frac{1}{\tilde{\tau}_{MF}} \frac{1}{\beta^{7/2}}. \qquad (33)$$

It is obvious that, in this special case, the Ω_C is very sensitive on the compact- ness parameter β. The most compact configuration of a neutron star leads to

dramatic lowering of the critical angular velocity values. For example when the value of β varies on the interval $0.1 - 0.2$ then (and for a the typical value $\tilde{\tau}_{MF}$=8 s) the Ω_C varies on the large interval 17076 – 1510 Hz. In addition, the combination of Eqs. (33) and (24)-(26) and considering that $M = 1.4\ M_{\odot}$ yields to a dependence of Ω_C on the parameter L, that is

$$\Omega_c \simeq (2757 \pm 492)\,\frac{1}{\tilde{\tau}_{MF}}\left(\frac{L}{\mathrm{MeV}}\right)^{7/8}(\mathrm{Hz}). \qquad (34)$$

In any case, it is worth pointing that according to the analysis of [14] only 2% of the expected range of ϵ leads to the time scale $\tilde{\tau}_{MF}$ shorter than 15 s in neutron stars with temperature about 10^8 K (that are typical for low mass x-rays binaries).

In the present work we investigated r-mode constraints from the neutron star equation of state. Firstly, we examined the case of a neutron star with a fluid interior and we derived an analytical solution for the Ω_C T dependence. In particular, we used a set of analytical solution of the TOV equations in or- der to reveal the role of the bulk neutron star properties (radius, mass, mass distribution) on the r-mode instability window. The main findings include the strong dependence of Ω_C on the neutron star size and the very weakly depen- dence on the other two properties for low values of temperature. Secondly, we examined the more realistic case where the effect of the solid crust is included in our study. In this case we found that the effect of the radius is also the most important but the dependence is more weakly compared to the fluid in- terior case. In any case, the dissipation effect due to the solid crust decreases considerably the instability window.

In view of the above results and motivated by the strong radius dependence of the critical angular velocity, we propose a phenomenological approach in order to correlate Ω_C with microscopic properties of the nuclear equation of state. This approach, despite of its simplicity, provides a few insights on the study of the effects of the EOS on the r-mode instability window, in a uni- versal way. In particular, the radius of a NS depends strongly on the specific character of the EOS for densities close to the saturation density. By employing an empirical relation, we related the Ω_C to the slope parameter L which is an individual characteristic of any EOS. We also proposed an approximated formula for the Ω_C L dependence applicable for a large number of EOS. This approach leads to some simplified empirical relations. Moreover, the proposed method provides, in a way, constraints on the nuclear equation of state with the help of accurate measurements of the main bulk neutron star properties. We also examined the case of an elastic crust via the slippage factor S. We found that this factor is the most important, concerning the estimation of the instability window. The measure of S is of importance, in order to define re- liable estimation of the corresponding instability window. On the other hand, we proposed possible measure of S in the case of accurate measures of Ω_C, R and T.

Finally, we verified previous studies that the mutual effects are very important and under some assumptions could explain the observation data, concerning old cold neutron star, even in the case of hadronic matter. However, more the- oretical work is appropriate in order to establish in details the mutual friction dissipations effects and to clarify further the equation of state constraints on the r-mode instability window.

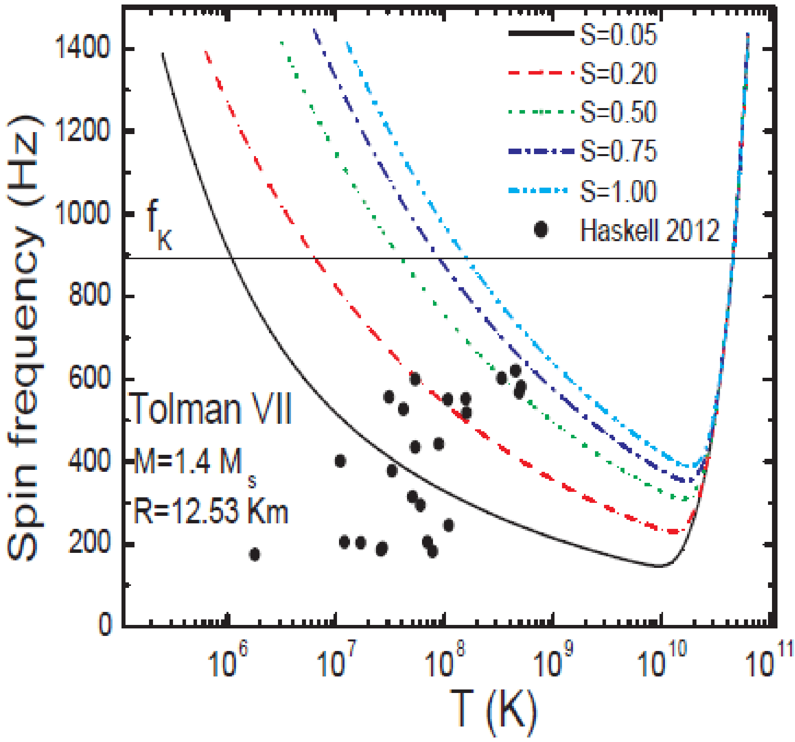


Fig. 1. The instability window for the Tolman VII solution when the elasticity of the crust is taken into account via the slippage factor *S*. The observed cases of LMXBs and MSRPs from [20] are also included for comparison.

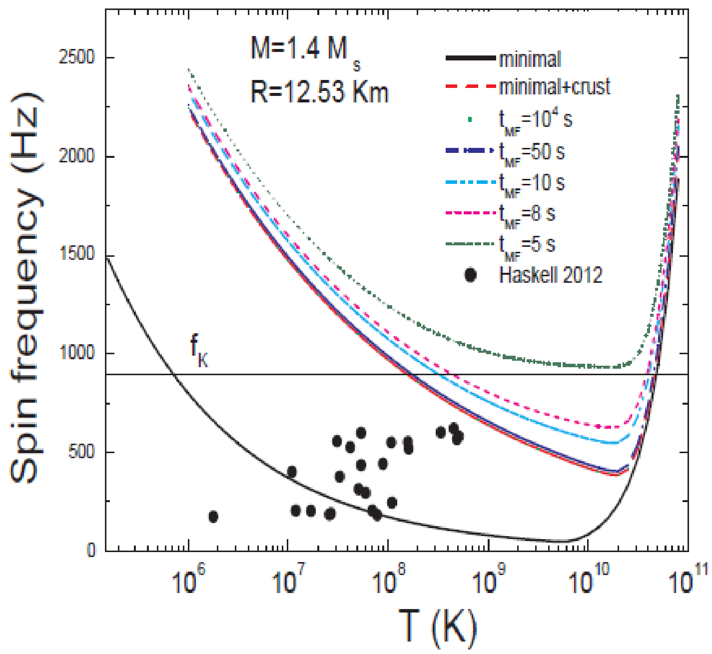


Fig. 2. The instability window for the Tolman VII solution for the cases a) minimal model, b) minimal model+crust considering slippage factor S = 1 and c) minimal model+crust including also mutual friction effects for various values of the time scale $\tilde{\tau}_{MF}$. The observed cases of LMXBs and MSRPs from [20] are also included for comparison.

References

[1] Papazoglou, M.C., Moustakidis, C.C., Astrophys Space Sci **316**, 98 (2016)

[2] Shapiro, S.L., and Teukolsky, S.A., *Black Holes, White Dwarfs, and Neutron Stars* (John Wiley and Sons, New York, 1983)

[3] Prakash, M., Nucl. Phys. **A 928**, 260 (2014)

[4] Lindblom,L., Owen, B.J., and Ushomirsky, G., Phys. Rev. D **62**, 084030 (2000a)

[5] Andersson, N., Astrophys. J. **502**, 708 (1998)

[6] Lindblom, L., Owen, B.J., and Morsink, S.M., Phys. Rev. Lett., **80**, 4843 (1998)

[7] Owen, B.J., Lindblom, L., Cutler, C., Schutz, B.F., Vecchio, A., and Andersson, N., Phys. Rev. D **58**, 084020 (1998)

[8] Moustakidis, Ch.C., Phys. Rev. C **91**, 035804 (2015)

[9] Bildsten, L., and Ushomirsky, G., Astrophys. J. Lett. **529**, L33 (2000)

[10] Andersson, N., and Kokkotas, K.D., Int. J. Mod. Phys. D **10**, 381 (2001)

[11] Levin, Y., and Ushomirsky, G., Mon. Not. R. Astron. Soc. **324**, 917 (2001)

[12] Kinney, J., and Mendell, G., Phys. Rev. D **67**, 024032 (2003)

[13] Glampedakis, K., and Andersson, N., Phys. Rev. D **74**, 044040 (2006)

[14] Lindblom, L., and Mendell, G., Phys. Rev. D **61**, 104003 (2000b)

[15] Lattimer, J.M., and M. Prakash, M., Phys. Rep. **442**, 109 (2007)

[16] Prakash, M., The Equation of State and Neutron Stars, lectures delivered at the Winter School held in Puri India, 1994 (unpublished)

[17] Lattimer, J.M., and Prakash, M., Astrophys. J. **550**, 426 (2001)

[18] Lattimer, J.M., and Lim, Y., Astrophys. J. **771**, 51 (2013)

[19] Demorest, P.B., Pennucci, R., Ransom, S.M., Roberts, M.S.E., and Hessels, J.W.T., Nature **467**, 1081

(2010)

[20] Haskell, B., Degenaar, N., Ho, W.C.G., Mon. Not. R. Astron, Soc. **424**, 93 (2012)

[21] Watts, A.L., Krishnam, B., L. Bildsten, L., and Schutz, B.F., Mon. Not. R. Astron. Soc. **389**, 839 (2008)

[22] Keek, L., Galloway, D.K., in't Zand, J.J.M., and Heger, A., Astrophys. J. **718**, 292 (2010)