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# A simple angle integration method for the determination of capture reaction cross sections

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**Abstract** In order to determine the cross section of a reaction by the angular distribution method, it is essential to calculate the angular distributions of all secondary transitions that lead to the ground state. This is achieved by analysing spectra from HPGe detectors at various detection angles, which is a time consuming process. In this work, a simpler angle integration method [1] is presented that allows to determine cross sections of capture reactions by measuring spectra at fewer detection angles.

**Keywords** angle integration, capture reactions

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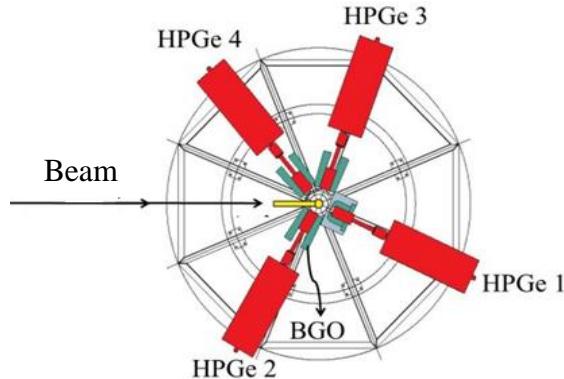
## INTRODUCTION

In capture reactions the compound nucleus decays from the entry state by the emission of gamma rays. The determination of the cross section of such a reaction is achieved by measuring the number of produced nuclei. This can be accomplished by measuring the primary gamma rays through which the entry state decays. However, the experimental determination of these gamma rays is not always possible, since capture reactions have high Q values, leading to entry states with high energy. As a result, the energies of the primary gamma rays are also high ( $> 6$  MeV), requiring detection systems with high efficiency. In addition, the decay of the entry state may occur through primary gamma rays which lead to high excited states, where usually not enough data are available. So, instead, in order to determine the number of the nuclei produced from the reaction, all secondary gamma rays which lead to the ground state are measured.

## CROSS SECTION MEASUREMENTS OF CAPTURE REACTIONS

### *Angular distribution method*

The angular distribution method is often used to determine the cross section of capture reactions. To do so, the angular distributions of all secondary gamma rays that lead to the ground state need to be measured. For this purpose, an experimental setup which consists of High Purity Germanium (HPGe) detectors is used. The detectors are placed on a goniometric table, in order to acquire measurements at various detection angles by the rotation of the table (Fig. 1).



**Fig. 1** Experimental setup for angle distribution measurements [2].

The determination of the total reaction cross section ( $\sigma_T$ ) with the angular distribution method is achieved by the use of the following equation

$$\sigma_T = \frac{A}{N_A \xi} Y \quad (1)$$

where  $A$  is the atomic weight of the target (g/mol),  $N_A$  is the Avogadro number (nuclei/mol),  $\xi$  is the target thickness ( $\text{g/cm}^2$ ) and  $Y$  is the reaction yield, namely the number of produced nuclei per beam particle. Even more specifically, if  $N$  transitions feed the ground state, the reaction yield is given by the sum over the coefficients  $(A_0)_i$ , as presented in the following expression.

$$Y = \sum_{i=1}^N (A_0)_i \quad (2)$$

The calculation of the coefficients  $(A_0)_i$  begins from the analysis of the experimental spectra and includes the subsequent steps:

- For each beam energy  $E_j$  and each detection angle  $\theta_k$  the collected experimental spectra are analysed for the determination of the area  $I$  of the desired photopeak.
- The area  $I$ , is corrected for the dead time and the absolute efficiency of the detector at angle  $\theta_k$ .
- The corrected areas are normalized to the beam charge and they are depicted in a graph as a function of the detection angle  $\theta$ , for each beam energy  $E_j$ .
- The determination of  $A_0$ , namely the absolute number of gamma rays with energy  $E_i$  which are emitted from the reaction in  $4\pi$ , is achieved by fitting the experimental data of the graph with the function

$$W(\theta) = A_0 \left[ + \sum_k a_k P_k(\cos \theta) \right], \quad k = 2, 4, \dots \quad (3)$$

where  $P_k$  is the Legendre polynomial of degree  $k$ .

As it has been so far presented, for the calculation of the cross section with the angular distribution method, measurements and analysis of experimental data at various detection angles are required. Thus, the process of obtaining the cross section with this method is time consuming. On the other hand, the cross section can be determined with the angle integration method [1] which requires measurements at fewer detection angles, simplifying this way the

measurement and analysis processes.

#### *Angle integration method*

The data analysis procedure for the determination of the cross section with the angle integration method was developed by Mihailescu et al. [1]. The first step is the experimental determination of the differential gamma ray production cross section ( $d\sigma_\gamma/d\Omega$ ) for all transitions that lead to the ground state and for all detection angles. Then with the use of the Gaussian quadrature method the angle integrated cross sections are calculated as it is presented in the following.

The differential cross section of a gamma ray transition is given as a finite sum of the even order Legendre polynomials as shown in equation (3). This equation can be integrated by the Gaussian quadrature method

$$\sigma_\gamma = 2\pi \int_{-1}^1 \frac{d\sigma_\gamma}{d\Omega}(x) dx = 2\pi \sum_{i=1}^n w_i \frac{d\sigma_\gamma}{d\Omega}(x_i) \quad (4)$$

where  $n$  is the number of detectors that are used for the experiment,  $w_i$  are the corresponding weights.

So, for a specified number of detectors the solution of the system gives the angles  $x_i$  and the weights  $w_i$  for which the sum gives the exact result to the integral. The solution of the system is accomplished when  $x_i$  equals to the root of the Legendre polynomial  $P_n(x)$  and the corresponding weights  $w_i$  are given in Table 1, depending on the number of detectors used. As conditioned by the Gaussian quadrature method the sum gives accurate results for polynomials with degree up to  $4n-2$ . The maximum degree polynomial that needs to be taken into account in order to describe accurately the angular distribution of a gamma ray transition depends on the spins of the nuclear states, the orbital angular momentum of the particles and the multipolarities of the gamma rays involved in the process [3].

Number of detectors	Weight	Angle
$n = 1$	2	55, 125
$n = 2$	0.696	31, 149
	1.304	70, 110
$n = 3$	0.936	76, 104
	0.722	49, 131
	0.343	21, 159

**Table 1** Weights and angles for the determination of the angle integrated cross section [1].

After having calculated all gamma ray production cross sections for all secondary gamma rays that lead to the ground state, the total reaction cross section for each beam energy is calculated with the following formula

$$\sigma_T = \sum_{i=1}^N \sigma_\gamma(L_i \longrightarrow g.s.) \quad (5)$$

where  $N$  are the number of transitions which lead to the ground state and  $L_i$  the gamma ray

production cross section for such a transition, as it has been calculated with the angle integration method.

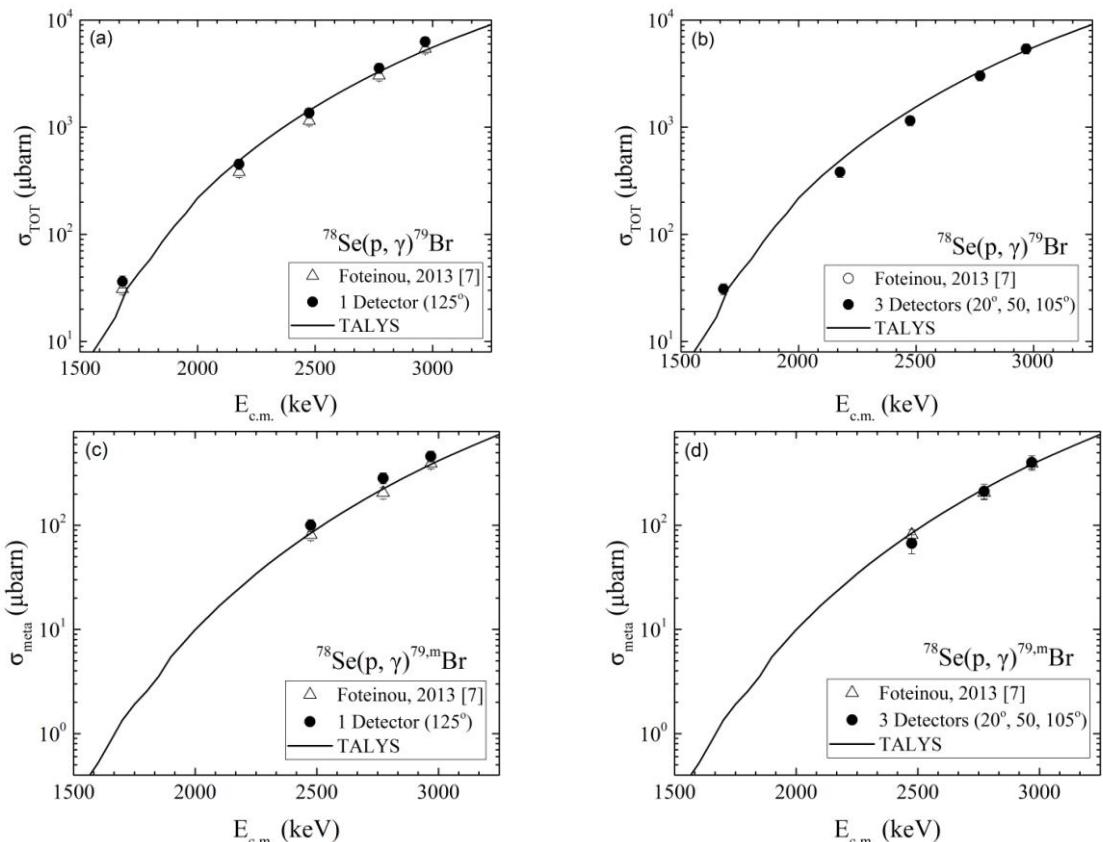
## EXPERIMENTAL MEASUREMENTS BY THE ANGLE INTEGRATION METHOD

### *Neutron inelastic scattering cross section*

Until today, cross section measurements of neutron inelastic scattering have been conducted with the angle integration method. For instance, the total neutron inelastic cross sections of the reactions  $^{52}\text{Cr}(\text{n}, \text{n}\gamma)$  [4],  $^{208}\text{Pb}(\text{n}, \text{n}\gamma)$  [5] and  $^{209}\text{Bi}(\text{n}, \text{n}\gamma)$  [6] have been measured at Geel Electron LINear Accelerator (GELINA). These measurements show good agreement with independent measurements for the same total neutron inelastic cross sections and with TALYS calculations.

### *Proton capture cross section*

In order to validate the results of the angle integration method in cross section measurements of proton capture reactions the nuclear astrophysics group of “Demokritos” used pre-existing angular distribution data for the reaction  $^{78}\text{Se}(\text{p}, \gamma)^{79}\text{Br}$  [7]. The method was applied for the cases of one and three detectors, placed at the angles suggested in Table 1. Then, the total cross section of the reaction  $^{78}\text{Se}(\text{p}, \gamma)^{79}\text{Br}$  as well as the cross section for the creation of the compound nucleus  $^{79}\text{Br}$  in its metastable state were compared to the angular distribution results [7] (Fig. 2).



**Fig. 2** Comparison between the angular distribution method [7] and the angle integration method

for the total cross section of the reaction  $^{78}\text{Se}(\text{p}, \gamma)^{79}\text{Br}$  and the cross section of the creation of the compound nucleus  $^{79}\text{Br}$  in its metastable state, with the use of one (a, c) and three detectors (b, d) for the angle integration calculation.

As it is shown in Fig. 2 the total and metastable cross section of the reaction  $^{78}\text{Se}(\text{p}, \gamma)^{79}\text{Br}$ , calculated with the angle integration method using three detectors, is in very good agreement with the results of the angular distribution method, as well as with TALYS calculations [8]. In this case, the two methods agree within error in all data points. On the other hand, when only one detector is used for the calculations the divergence of the two methods is larger. This suggests that using Legendre polynomials with degree up to 2, as results from the  $4n-2$  rule of the Gaussian quadrature method, in order to describe the angular distribution of the gamma rays of this reaction is not sufficient and higher degree polynomials need to be added to the calculation.

## RESULTS AND DISCUSSION

The angle distribution method is often applied for the measurement of proton capture reaction cross sections. The method requires measurements at many detection angles. As a result, the experiment and data analysis process are time consuming. On the other hand, in order to obtain the same quantity with the angle integration method, measurements at fewer detection angles are required, simplifying this way the experimental and analysis processes. The nuclear astrophysics group of “Demokritos” compared the results of the angle integration method using one and three detectors with the results of the angular distribution method obtained from the same experimental data. The two methods presented very good agreement in the case where three detectors were used in order to obtain the cross section. However, when one detector was used for the calculations the differences between the two methods were higher, suggesting that Legendre polynomials of greater degree need to be taken into account to the calculation.

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