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*Patroklos-Miltiadis Mamaloukas, Theo J. Mertzimekis*

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# A systematic analysis of nuclear charge radii data

Patroklos-Miltiadis Mamaloukas and Theo J. Mertzimekis

*Department of Physics, University of Athens, Zografou Campus, GR-15784, Athens, Greece*

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**Abstract** Driven by the need to describe strong nuclear interactions between hadrons in a nucleus, we take a closer look at evaluated experimental data of nuclear charge radii acquired mainly with laser spectroscopy methods. Understanding the range at which the components of the nucleus interact with each other shall lead to even more accurate models of nuclear potentials, and even understanding limitations of fundamental symmetries. The present work takes advantage of the existing IAEA database by Angeli & Marinova [1], and an additional set of recently surveyed measurements, specifically between 2013–2018. Statistical treatment includes different polynomial fits for the entirety of the data set with the intention of confirming or improving the well-established empirical formula,  $R=r_0A^{1/3}$ . Various trends of observed deviations from this formula are investigated in terms of structure effects spanning major shells and near magic numbers.

**Keywords** nuclear charge radius, nuclear data

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## INTRODUCTION

The term hyperfine shift refers to small shifts in the energy levels of atoms, and with energy shifts typically orders of magnitudes smaller than those of a fine-structure shift, results from the interactions of the nucleus with internally generated electric and magnetic fields [1]. It will be our tool of measuring the (rms) charge radius of the nucleus, i.e. the average radius that the charge inside the nucleus is bounded. Across the isotopic chart, trends of nuclear charge radii data can reveal information on the fundamental interactions shaping the shape of atomic nuclei.

## THEORETICAL BACKGROUND

It is widely accepted that the strong force displays isospin symmetry, i.e. there is no distinction between positively charged nucleons (protons) and neutral nucleons (neutrons). Additionally, the strong force, in contrast with electromagnetism and gravity, exhibits finite range, a property that limits its effective range to a maximum value of about 10 fm. Precisely defining that maximum value is crucial for any model simulating a realistic nuclear potential, as well as determining its evolution across all mass regimes.

The condition upon which we shall approach the radius problem is considering that nucleons, instead of directly interacting with each other, react in a mean field produced by the sum of every individual interaction. As a direct consequence, description by a mean field implies spherical symmetry of the nucleus, thus making the measurement of the nuclear charge radii a direct determination of the nucleus size.

As far as the nuclear radius is concerned, both experiments and theory seem to converge on the formula  $R=r_0A^{1/3}$  where  $r_0$  is a phenomenological parameter and  $A$  is the mass number

of each isotope [2]. This formula is expected to provide accurate predictions especially near magic numbers, for which we observe energy shell closure based on the mean field shell model, thus ensuring a spherical shape for these nuclei.

## NUCLEAR DATA AND DATA ANALYSIS

Several experimental data sets exist for nuclear charge radii accumulated with a variety of experimental techniques, such as scattering or atomic spectroscopies. In particular for hyperfine shifts the most widely used ones lately depend on laser spectroscopy (e.g. collinear laser spectroscopy). Due to the hyperfine interaction between the nucleus and the surrounding electron cloud, information about the nucleus can be obtained by manipulating the atomic electrons. Laser light with a very precise wavelength is used to induce electron-transitions in atoms or ions, and from the hyperfine splitting (HFS) or isotope shifts (IS), the nuclear spins, moments and charge radii can be extracted [3]. In an alternative approach, the ion beam can be prepared and delivered to the experiment as in collinear laser spectroscopy. The atom beam is then overlapped with additional laser beams for efficient ionization. The ionized beam is finally deflected towards a high-efficiency counting station.[4]

In the present work, a statistical treatment of nuclear charge radii has been performed. Data were retrieved from:

- Set #1: The compilation of nuclear charge radii by Angeli and Marinova [5], a cumulative paper of charge radii measurements made from 1964 up to 2012.
- Set #2: An exhaustive survey of recent literature after 2012 [6–23] was performed and data were retrieved mainly from peer-reviewed journals and conference proceedings.

From the data sets, several plots have been constructed:

- The average nuclear charge radius in relation to  $A$  from [5] including polynomial fits
- The average nuclear charge radius in relation to  $A$  from [6–23] including polynomial fits
- A cumulative plot of nuclear charge radii in relation to  $A$  including polynomial fits

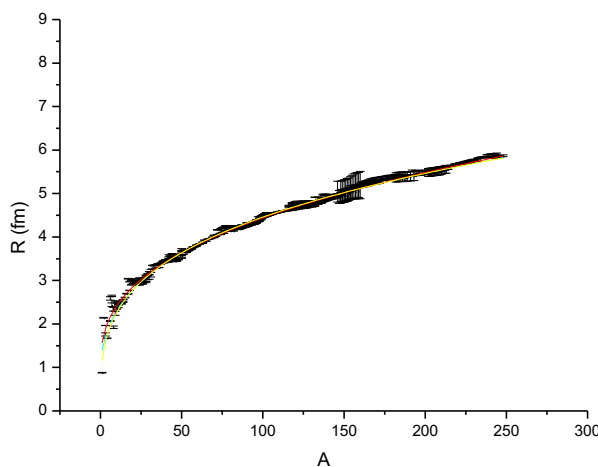


Figure 1 A cumulative plot of nuclear charge radii with respect to the mass number  $A$ . The solid line is a polynomial fit.

A function of the form  $R=r_0A^C+B$ , with parameters  $r_0$ ,  $B$  and  $C$ , was used to fit the data in all cases. The fit on all data resulted in  $r_0=0.6153(213)$  fm,  $C=0.3768[48]$ , and  $B=0.9509[374]$

fm. The exponent  $C \approx 0.38$  is close to the value  $1/3$  ( $\approx 0.33$ ), but statistically different. Also,  $r_0$  is not in the typical range of 1.2–1.4 fm, which is commonly found in textbooks. Further investigation showed that this deviation can be attributed mainly to the fit at high  $A$ , thus implying the departure of heavy nuclei from sphericity. Additional graphs have been prepared to examine trends across the full span and in isolated mass regimes of the isotopic chart:

- The difference between theoretically expected value and experimental result for each  $A$  in relation to  $A$  from [5]
- The difference of the average radius between  $A$  and  $(A-1)$  isotope with respect to  $A$
- The Gaussian behaviors for three individual regimes in the range  $A=140$ – $200$  to examine nuclear shell effects on the nuclear charge radius
- The actual deviation of experimental data from theory for all isobars

## CONCLUSIONS

Polynomial fits on all the available data both [5] and [6]–[23] seem to almost converge on the semi-empirical formula  $r=R_0A^{1/3}$  for nuclear radius, part for a slight preference of the nuclear charge to be more tightly packed than the total of nuclear mass (comparing exponentials:  $0.33 < 0.38$ ). Moreover, we observe the greatest of deviations from the theoretical model around  $A=168$ , which is a magic number in the harmonic oscillator model. Finally, isospin symmetry seems to be slightly violated given the non-linear behavior for some of the diagrams. Further investigation is required to have a firm picture of the structure effects in nuclear charge radii.

## References

- [1] A.R. Bodmer, Nucl. Phys. 9, 371 (1958/59)
- [2] L.R.B. Elton, Nucl. Phys. 5, 173 (1958)
- [3] COLLAPS Collaboration, ISODLE/CERN, url: <https://collaps.web.cern.ch/>
- [4] T.E. Cocolios et al. NIM B 317, 565 (2013)
- [5] I. Angeli and K.P. Marinova, Atomic Data Nuclear Data Tables 99, 69 (2013)
- [6] K. Kreim et al., Phys. Lett. B 731, 97 (2014)
- [7] D.M. Rossi et al., Phys. Rev. C 92, 014305 (2015)
- [8] G.J. Farooq-Smith et al., Phys. Rev. C 96, 044324 (2017)
- [9] H. Heylen et al., Phys. Rev. C 94, 054321 (2016)
- [10] A. Voss et al., Phys. Rev. A 95, 032506 (2017)
- [11] A.E. Barzakh et al., Phys. Rev. C 94, 024334 (2016)
- [12] M.L. Bissell et al., Phys. Rev. C 93, 064318 (2016)
- [13] R.F. Ruiz-Garcia et al., Nature Physics 12, 594 (2016)
- [14] D.A. Fink et al., Phys. Rev. X 5, 011018 (2015)
- [15] M.D. Selistersov et al., Phys. Lett. B 719, 362 (2013)
- [16] Y. Hirayama et al., Phys. Rev. C 96, 014307 (2017)
- [17] K.M. Lynch et al., Phys. Rev. X 4, 011055 (2014)
- [18] K. Minamisono et al., Phys. Rev. Lett. 117, 252501 (2016)
- [19] A. Ozawa et al., Phys. Rev. C 89, 044602 (2014)
- [20] A.E. Barzakh et al., Phys. Rev. C 95, 014324 (2017)
- [21] D.T. Yordanov et al., Phys. Rev. Lett. 116, 032501 (2016)
- [22] K. Tsukada et al., Phys. Rev. Lett. 118, 262501 (2017)
- [23] R. Pohl et al., Science 353, 669 (2016)