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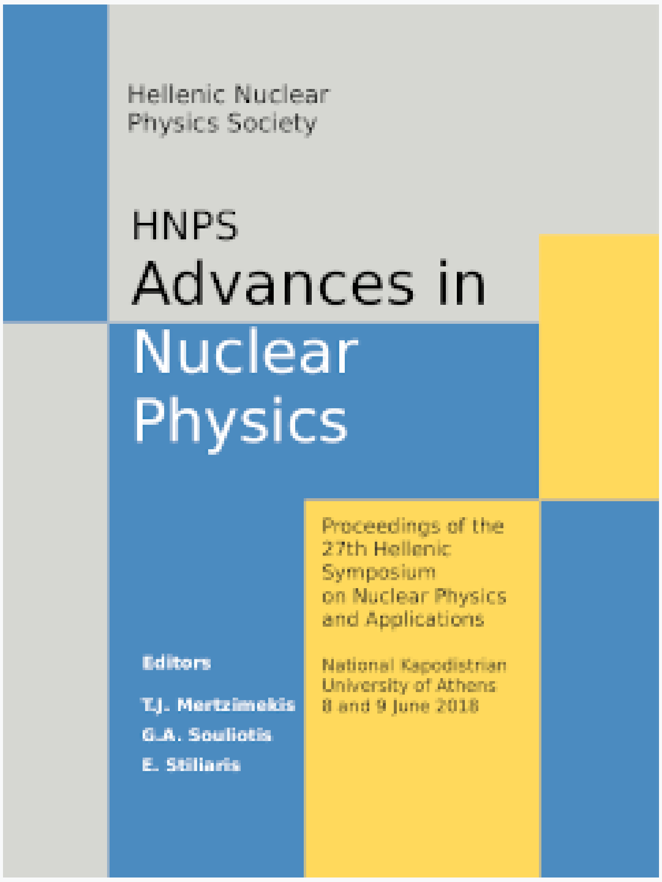

Equation of state effects on the core-crust interface of slow rotating neutron stars

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Equation of state effects on the core-crust interface of slow rotating neutron stars

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Abstract We systematically study the symmetry energy effects of the transition density n_t and the transition pressure P_t around the crust-core interface of a neutron star in the framework of the dynamical and the thermodynamical method respectively. We employ both the parabolic approximation and the full expansion, for the definition of the symmetry energy. We use various theoretical nuclear models. Firstly we derive and present an approximation for the transition pressure P_t and crustal mass M_{crust}. Secondly, we explore the effects of the Equation of State (EoS) on a few astrophysical applications which are sensitive to the values of n_t and P_t. We found that the above quantities are sensitive mainly on the applied approximation for the symmetry energy (confirming previous results). Furthermore, an additional sensitivity also exists, depending on the used method (dynamical or thermodynamical). The above findings lead us to claim that the determination of the n_t and P_t must be reliable and accurate before they are used to constrain relevant neutron star properties.

Keywords Neutron stars, Nuclear equation of state, Crust-core interface, Dynamical method

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INTRODUCTION

It is known that for low densities (close to the saturation density of symmetric nuclear matter) the EoS is well constrained and the relevant predictions are more reliable. This prediction includes the crust-core interface of a neutron star which is the main subject of the present study. The equation of state of neutron-rich nuclear matter is the important ingredient in the study of both the core and the crust, of all the bulk properties of a NS.

The motivation of the present work is twofold. Firstly, in the framework of the dynamical and the thermodynamical method we calculate the transition density and the corresponding pressure using various nuclear models. Secondly, we concentrate our study mainly on the error which can be introduced by employing the well known parabolic approximation for the symmetry energy, not only on the values of n_t and P_t but also on the predictions of some neutron star observable properties. We exhibit the necessity to implement both the dynamical method and the full expansion for the symmetry energy in order to get reliable predictions. Moreover, we provide expressions for the mass of the crust M_{crust} and also for the transition pressure P_t. Finally, we employ the Tolman VII analytical solution of the TOV equations and we derive expressions for the time scales and frequencies related with the r-mode instabilities (which are sensitive on the crust-core interface).

DYNAMICAL METHOD FORMALISM AND SYMMETRY ENERGY

The study of the instability of β stable nuclear matter is based on the variation of the total energy density, in the framework of the Thomas-Fermi approximations [1]. In the dynamical (DYN) method, compared to the thermodynamical (THER) one, effects from inhomogeneities of the density and the Coulomb interaction have also been included. In the framework of this method, the so-called effective interaction between protons $U_{dyn}(k,n)$ is given by an expression that, for small values of the momemtum wave-number k and by neglecting the factor D_{ee} in the full expression, leads to the well known approximation [1]

$$U_{dyn}(k,n) = U_0(n) + \xi k^2 + \frac{4\pi e^2}{k^2 + k_{TF}^2}\,.$$

The gradient terms D_{ij} (i, j = p, n) that appear in the above expression are, in general, functions of the density but we treat them as constants. Moreover, since the models used in the present work do not have gradient terms we fix them in an approximated way in order to get reasonable values for the surface thickness $t_{90\text{-}10}$ and the surface tension σ_{snm}. The effective interaction $U_{dyn}(k,n)$ for a fixed value of the density n, has a minimum at k = Q. The transition density n_t is determined from the condition $U_{dyn}(Q, n_t) = 0$. For more details see also [2].

The symmetry energy plays an important role on the determination of the transition density and the corresponding pressure and is a key quantity to explain in general many neutron star properties and dynamical processes. We consider that the energy per particle of nuclear matter E_b can be expanded around the asymmetry parameter I [3]. The coefficient of the quadratic term of this expansion is the nuclear symmetry energy E_{sym} and the slope of the symmetry energy L at the nuclear saturation density n_s, which is an indicator of the stiffness of the EoS, is defined as the derivative of E_{sym} [2]. In β-stable nuclear matter, due to chemical equilibrium, it is easy to find a general relation that determines the proton fraction. We are referring to this relation as the full expansion (FE). Having this, we find the relation for the transition pressure P_t^{FE} in the case of the FE [2]. Accordingly, in the framework of the parabolic approximation (PA), we find a relation for the transition pressure P_t^{PA} [3].

THE NUCLEAR MODELS

In the present work we employed various nuclear models, which are suitable to reproduce the bulk properties of nuclear matter at low densities, close to saturation density as well as the maximum observational neutron star mass. The nuclear models that we use are the following: the momentum-dependent interaction model (MDI model), the HLPS model and the well-known Skyrme parametrization.

APPLICATIONS

We provide some applications of the crust-core interface in astrophysics. In particular we concentrate our study on the effects of the transition density and transition pressure on a) the radius and mass of the crust (one can see similar studies in [4]), b) the oscillation frequencies obtained from observations of quasi-periodic oscillations (QPOs) in X-ray emissions which are caused most likely due to torsional vibration of the crust of a neutron star (for more details see the discussion in [5]), c) the thermal relaxation time of the crust during the cooling process of a hot neutron star [5,6], d) the crustal fraction of the moment of inertia and its effects on the creation of neutron star glitches ([7] and references therein) and e) the conditions for the r-mode instabilities of rotating neutron stars [8,9].

RESULTS AND DISCUSSION

Firstly, we check the accuracy of the approximation for the effective interaction $U_{dyn}(k,n)$ compared with the full expression (the results and conclusions are similar for all the employed nuclear models). We found that using the full expression for the dynamical potential, in each case, the accuracy of the transition density is better than 0.5% while for the transition pressure is better than 1%. Similarly, using the parabolic approximation for the symmetry energy, the accuracy of the approximation (compared to the full expression), concerning the values of n_t and P_t, is also better than 0.5% and 1% respectively. It is worth to notice that the values of D_{ij} are not included in the nuclear models and must be inserted artificially. For reliable values of D_{ij}, both relations predict similar results.

Now, according to the discussion above, we use mainly four cases. Firstly we employ the dynamical method by considering for the calculation of the proton fraction the general relation (DYN-FE case hereafter) and the one that is produced by the use of the parabolic approximation (DYN-PA case hereafter). Secondly, we employ the thermodynamical method accordingly, so we have two more cases (THER-FE case and THER-PA case hereafter). The use of the dynamical method, in the framework of DYN-FE, significantly lowers the values of n_t and P_t compared to the use of THER-PA or THER-FE.

It is worth to discuss that the approximations for the crustal radius and the corresponding mass, which are introduced in the present study, are very accurate and are used to derive some analytical expressions for the thermal relaxation time, the QPOs frequencies and the critical angular velocities. In the present work we derived also a semi-theoretical expression which relates the transition pressure P_t with the total radius R of a neutron star with 1.4 solar masses. Since the majority of the observational neutron stars has a mass close to this limit, the accurate observational measurement of R may help to constraint P_t and vice versa. Another semi-theoritical expression between P_t and L is discussed further. This expression is satisfied by the THER-FE and DYN-FE cases. However, the use of the parabolic approximation leads to the inverse behavior.

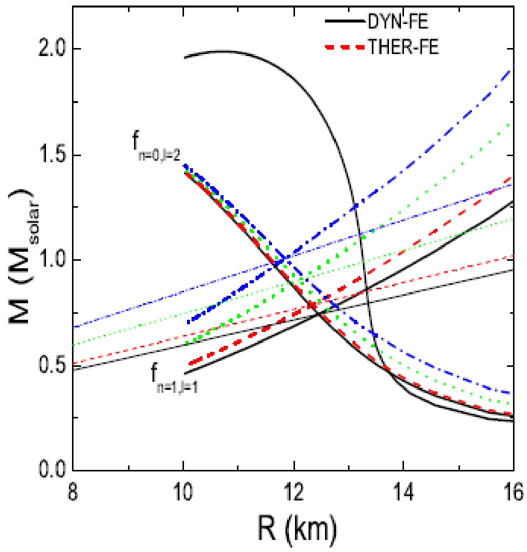

Fig. 1. The mass as a function of the radii for the MDI model (for L = 80 MeV) including constraints from neutron star seismology for the four selected cases.

In Fig. 1 we present a mass-radius diagram showing the constraints from neutron star seismology (originated from the soft gamma-ray repeater SGR 1806-20) [5]. Specifically, we plot the mass-radius dependence using the MDI model (L=80 MeV). We consider that $f_{n=0,l=2}$= 29 Hz and also $f_{n=1,l=1}$= 626.5 Hz [5] and we solve the corresponding equations for four selected cases. Obviously the effects of the transition density are more pronounced in the case of the $f_{n=1,l=1}$ modes. As a general conclusion the use of a more realistic EoS (DYN-FE) decreases appreciably the values of the mass. In the case of the fundamental mode $f_{n=0,l=2}$, the effects of the EoS are less important and appear mainly for high values of the radius. Furthertmore, the use of DYN-FE leads to dramatic lowering of the values of t_w. It is very interesting to see that the use of THER-PA leads to very high values of t_w (more than twice for low masses) compared to the DYN-FE case and consequently to a huge error.

The effects of the transition pressure P_t and density n_t are important also on the calculations of the crustal moment of inertia. Actually the major dependence is upon the pressure P_t. As we can see from Fig. 2, the use of the DYN-FE model (which leads to low values of P_t) decreases the allowed region compared to the other three cases.

There is a moderate dependence of the critical frequency on n_t and P_t. In particular, the use of the DYN-FE method increases the instability window and consequently increases the possibility few neutron stars to be, via the r-mode instability, sources of variational waves.

CONCLUSIONS

The main motivation of the present work is not only to study in detail possible constraints on the EoS from the crust-core interface since many studies have been dedicated to this effort. Actually we intended also to exhibit the extent of sensitivity of the EoS constraints on the crust-core interface properties (density, pressure, chemical potential e.t.c). We focused on the effects of the error which is introduced by employing the parabolic approximation in the framework of the dynamical and thermodynamical method. We estimated that although the PA is an accurate approximation for the total energy per baryon of nuclear matter, its derivative (which is involved in the calculations of n_t and P_t via the symmetry energy) is not. Consequently the deviations from the use the FE are important and must be taken into account. In total, our findings support the statement that the crust-core interface point must be estimated with a high accuracy so that the imposed constraints on the EoS can be as much as possibly reliable.

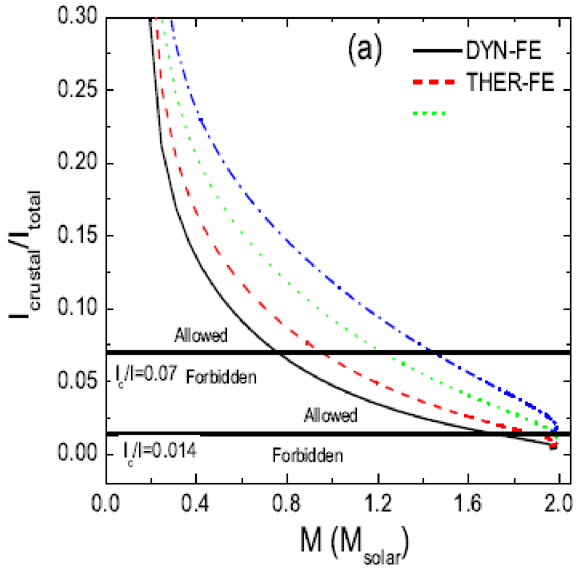

Fig. 2. The crustal fraction of the moment of the inertia as a function of mass presented for the four selected cases. For comparison we include the horizontal lines, each one representing a possible I_{crust}/I constraint.

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References

[1] G. Baym et al., Nucl. Phys. A 175, 225 (1971)
[2] L. Tsaloukidis, Ch. Margaritis, and Ch.C. Moustakidis, arXiv:1803.06633
[3] Ch.C. Moustakidis, Phys. Rev. C 86, 015801 (2012)
[4] J.L. Zdunik et al., A&A 599, A119 (2017)
[5] J.M. Lattimer and M. Prakash, Phys. Rep. 442, 109 (2007)
[6] O.Y. Gnedin et al., Mon. Not. R. Astron. Soc. 324, 725 (2001)
[7] B. Link, Astrophys. J. 789, 141 (2014)
[8] L. Lindblom et al., Phys. Rev. D 62, 084030 (2000)
[9] Ch.C. Moustakidis, Phys. Rev. C 91, 035804 (2015)