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A moving bead on a rotating rod

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Abstract

We present a cross thematic activity in the fields of physics and mathematics. This is part of a series of three activities in which students are guided to conjecture or discover the behaviour of physical models and express physical laws using mathematics from their school curriculum. They use The Geometer's Sketchpad dynamic geometry environment to simulate, verify and utilize their results or to hypothesise on the theoretical properties of a moving bead on a rotating rod. A thematic design triangle that takes into consideration the two basic axes of computer supported collaborative learning and the notion of a scientific model was used for the cross thematic approach.

Keywords: cross thematic activity, geometer's sketchpad, young scientist, simulation, scientific model

Introduction

During the academic year 2009 – 2010, three cross thematic activities in the fields of physics and mathematics were carried out in the small private school "xy+ 1" on the island of Rhodes, Greece. Three eager and promising young students of the 1st year of high school volunteered to be guided through physical notions that were entirely or almost new to them as well as to relate the relevant physical models to mathematics of their school curriculum. They used the familiar to them dynamic geometry emulator The Geometer's Sketchpad in Greek (GSP) (Manual, 2000) for the ruler and compass constructions as well as the physical motion simulations. The three activities constitute an initial study which aims to provide a basis for more systematic data collection in future research. The first activity, "simulating Newtonian free fall by the use of Euclidean geometry", precedes the other two and has been published in detail (Pipinos, 2010), while the second activity, "constructing the centroid of non regular bodies" is to be presented in a conference next year. This work in progress is part of an ongoing effort of the author to explore the ability of students to learn by a performance that resembles original scientific research (Pipinos, 2007).

In this paper we report on the third activity, "a moving bead on a rotating rod". The students use familiar mathematics to encounter polar coordinates for the first time (without knowing that they do so). They also provide a mathematical condition that the simulated composite physical motion is periodic in quite an elementary way. ICT plays a central role in this activity, as it is used both to analyze the polar coordinate equation of motion that naturally arises from the physical setup itself, as well as to verify the derived condition for periodicity by creating controlled 'flower' curves. Duration, 2 ½ hours.

Our thematic design triangle

In order to design and carry out these cross thematic activities, we considered the theory of computer supported collaborative learning (CSCL) in conjunction with contemporary applications of the notion of a scientific model (Figure 1).

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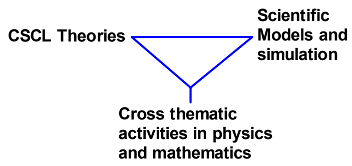

Figure 1. The three activities are based on this thematic design triangle

Computer supported collaborative learning

CSCL "is focused on how collaborative learning supported by technology can enhance peer interaction and work in groups" (Lipponen, 2002). A list of main such models can be found in Karasavvidis, 2006. Hakkarainen and his colleagues at the University of Helsinki, in their pedagogical 'progressive inquiry' model, propose didactic phases that resemble actual scientific research: setting up the context, presenting research problems, creating working theories and critical evaluation (Muukkonen et al., 2004). Similar concepts can be found in the model of 'knowledge building' (Scardamalia & Bereiter, 2003). In the 'knowledge integration' model, interactive dialogue is predominant in all of its design patterns (Linn, 2006). The teacher can integrate didactic activities in the 'knowledge creation' framework of learning, in the sense that "collaborative activities are organized around shared objects rather than take place through immediate interaction between participants (Lipponen et al., 2004).

Scientific models

The didactic CSCL theories described above are very closely related to student group work around a simulation of the real world. "Models are vehicles for learning about the world. Significant parts of scientific investigation are carried out on models rather than on reality itself because by studying a model we can discover features of and ascertain facts about the system the model stands for" (Hartmann & Frigg, 2006). Hestenes analyzes the components of a mathematical model, a scientific theory and the procedural knowledge of the scientific method from a pedagogical point of view (Hestenes, 1987). He describes four types of model structure (Hestenes, 1996) among which quite relevant to our activities are the components: configuration (geometric relations among the parts) and descriptive models (they represent change by explicit functions of time). Teachers around the globe incorporate the Modelling Instruction Program in workshops (Schober, 2003).

Cross thematic approach

The Pedagogical Institute (Π.Ι.) uses a cross thematic approach to knowledge in order to "encourage the interconnection of cognitive disciplines through appropriate extensions of taught subjects" via its Cross Thematic Integrated Curriculum Frame (Alachiotis, 2003), while "the aid of a computer and the appropriate dynamic simulations can prove very useful to the student so that they can apprehend and better understand concepts and procedures" (Pedagogical Institute, 2003).

A moving bead on a rotating rod

The students were instructed to construct a free point P rotating on a fixed circle (O, R) (with constant angular velocity ω) and the diameter from that point in order to simulate a rotating rod of length l. Then, they created another free point A on the rod that was to travel back and forth along the rod at a constant velocity v. Thus, we had a model of a moving bead along a rotating rod. By using the command 'trace' on the point on the rod and by creating an 'action button/animation' for the simultaneous motion of the rod and the point on it, the students were able to directly observe the flowerlike curves that emerged for various values of the two velocities.

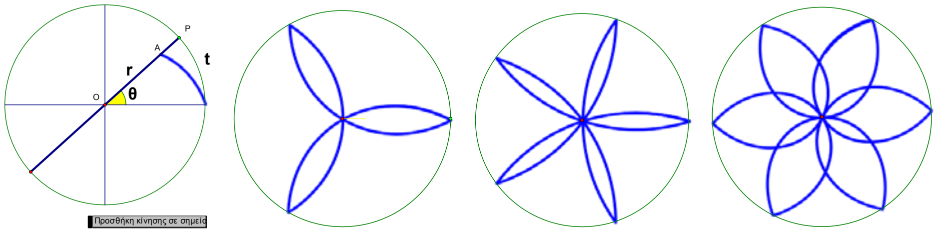

Figure 2. Deducing the polar equation of the curve and (1, 3), (1, 5), (2, 3) flowers are created by using the derived periodicity condition

When asked to find a way to determine the equation for the points of the curve, it was no surprise that they mentioned the usual Cartesian coordinates from the school curriculum. However, with little encouragement and discussion on why it is natural here to consider distance $r = OA$ and angle θ at time t, they quickly deduced the parametric polar equation $r = R - vt,\ \theta = \omega t$ that naturally led to the polar $r = R - \frac{v}{\omega}\theta$ by eliminating time t. We verified this equation (which holds until the moving point on the rod travels a half rod length) by the 'graph/plot new function/equation/ $r = f(\theta)$' tool of GSP.

Now the students were introduced to a challenge: to find a mathematical condition for our motion to be periodical in the sense that both free moving points must coincide at the original starting point at some future time. Working together, the students decided that periodicity means that "both the rod and the bead have completed a number of independent complete moves" but no mathematics had been written down yet. So we had to help them consider that the rod will have completed n turns and the bead m rod lengths at coincidence time t. We actually created a table for various values of n, m. They came up with necessary and sufficient equations $\theta = \omega \cdot t = \pi \cdot n$ and $v \cdot t = m \cdot l$. (There are some details in that n, m must have the same parity, but let us not get into this for didactic reasons. Besides, without this detail, periodic motion will occur in another sense anyway). It is notable that they derived these equations themselves after the teacher lead them to 'consider' n, m. By eliminating t again, a condition for periodicity is found: $\omega \cdot l / \pi \cdot v \in \mathbb{Q}$! Moreover, if u is the circular velocity of point P then this condition becomes $u/v = \pi/2 \cdot n/m$, i.e. a rational multiple of $\pi/2$. This enabled us to simulate any desired periodic 'flower' by setting the right ratio of velocities in submenu 'properties/animate/speed/other' of our animation button for corresponding desired values of n, m (Figure 2). Interesting closed shapes and other flowers occur for other values for n, m like $(2, 3)$ or $(3, 1)$. The students extremely enjoyed the fact that scientific considerations enabled them to "combine my favourite subjects (i.e. physics and geometry) to totally control the motion, creating something beautiful, when in the beginning it was just chaotic", as one of the students notes. They also realized a connection between rational numbers and periodicity. This connection appears in mathematics at university level when one considers when the sum of two periodic functions is periodic (Olmsted & Townsend, 1972) and at several other instances.

Conclusions

The presentation of the problem in itself has a vast impact for the rest of any activity. It is important to rely on the imagination, natural curiosity and thirst for discovery of our students. Dialogue, discussion and verbal communication form the basis of any effective pedagogical approach.

As teachers, we found that students were able to produce original ideas on which to work on either as a group or on their own. From their performance, comments and impressions we conclude that the cross thematic approach of simulating physical models based on an ICT setup can help them visualise better, invent laws and conditions, create geometry that works, and even get rid of previous misconceptions.

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