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Communicating recent customers’ decisions to strategic customers of a service system: the join or balk dilemma

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Abstract

In the present paper, we consider the fundamental model of Rational Queueing which concerns the join-or-balk dilemma of homogeneous strategic customers at the single-server Markovian queue with infinite waiting space. This model has been extensively studied under various assumptions regarding the information that is available to the customers upon arrival. The information assumptions that have appeared in the literature deal mainly with the possibility of the customers to observe the queue length before making their decisions (observable model, unobservable model, partially observable model, observable with delay model etc.). In the present paper, we introduce a new class of models where the information that is communicated to the arriving customers concerns the recent customers' decisions. We present various models that belong to this class and report some preliminary promising results that show that this kind of information is valuable and can lead to good outcomes.

JEL Classifications: C690, C720.

Keywords: service systems, queueing, strategic customers, equilibrium customer strategies, information

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1. Introduction

Rational Queueing is the branch of Queueing Theory that focuses on the game-theoretical analysis of service systems. The basic assumption of Rational Queueing is that the various agents (customers and/or administrators-servers) of a service system are strategic, i.e. they make decisions with the objective of maximizing their own utility function that represents their desire for service and their dislike for waiting. These ideas started about 50 years ago with the pioneering papers of Naor (1969), and Edelson and Hildebrand (1975) who studied the join-or-balk dilemma of arriving customers at the single-server Markovian queue with infinite waiting space. Naor (1969) focused on the observable model, where the arriving customers have the possibility to observe precisely the queue length before making their decisions, whereas Edelson and Hildebrand (1975) considered the unobservable counterpart, where the decisions are based solely on the economic and operational parameters of the system.

The research in Rational Queueing has been expanded considerably ever since. The monograph by Hassin and Haviv (2003) summarizes the main methodological tools in the area and fundamental models. Stidham (2009) and Hassin (2016) monographs are also devoted to the presentation of methods and results for this branch of Queueing Theory.

A fundamental issue in Rational Queueing is the impact of the information that is provided to the customers. The importance of this issue has been recognized by various studies and the interested reader may consult the recent reviews by Hassin (2016) (chapter 3), Ibrahim (2018), Economou (2021) and Economou (2022). More details are given in the literature review in section 2.

Most papers regarding the influence of the information on strategic customer behavior in service systems consider the number of present customers as the key information that is provided to the customers. However, this kind of information may not be available in practice. One such case occurs when there are independent web-based systems that receive the arriving customers that do not have information about what is going on in the core service system. This happens frequently when petitions for service are deposited through a web-platform and the service consists of several stages that are not monitored by the platform. In such a case the platform can provide information about previous arrivals and their join decisions but not about the actual congestion.

The present paper aims to introduce a family of models that deal with the information that the customers may receive about previous customers' decisions. The models are built for the situation of the join-or-balk dilemma of strategic customers in the M/M/1 queue (unobservable M/M/1 model of Edelson and Hildebrand (1975)).

The paper is structured as follows: In Section 2 we present a brief literature review regarding the problem of information provision to strategic customers of a queueing system. In Section 3, we present the main hypotheses concerning the study and, in Section 4, we describe the framework for the study of strategic customer behavior in a service system and then adapt the framework for the study of the join-or-balk dilemma for the arriving customers at a queue. In Section 5, we describe in detail several types of models regarding the information that can be provided to the customers about the decisions of previous customers and discuss their performance evaluation under arbitrary strategies. In Section 6, we present some preliminary analytical results on a simple information case. In Section 7, we show several numerical results that illustrate how this simple information case compares with the classical unobservable and observable models. The study finishes with a list of the results in Section 8 and with conclusions and directions for future research that are presented in Section 9.

2. Literature review

The literature that focuses on the effect of information on strategic customer behavior in service systems is very extensive. Two key references are the pioneering papers of Hassin (1986) and Chen and Frank (2004) who compared the equilibrium performance of the observable and unobservable versions of the single-server Markovian queue with strategic customers who face the join-or-balk dilemma. These papers showed that it is advantageous in some cases to reveal the queue length and in other cases to conceal it.

Various authors considered models that lie between the two extreme information versions (observable and unobservable) of the above situation. More specifically, the following categories of models have appeared in the literature (see Economou (2021) and Economou (2022)):

- *Systems with imperfect observation structure.* In such systems, the customers receive imperfect information about the queue length (see e.g., Economou and Kanta (2008), Guo and Zipkin (2009), and Hassin and Koshman (2017)).
- *Systems with delayed observation structure.* In such systems, the customers observe the queue length with some delay (see e.g., Burnetas, Economou and Vasiliadis (2017), and Hassin and Roet-Green (2020)).

- *Systems with mixed observation structure.* In such models, only a fraction of the customers observe the queue length (see e.g., Economou and Grigoriou (2015), and Hu, Li and Wang (2018)).
- *Systems with alternating observation structure*. Under such an information structure, a system alternates between observable and unobservable periods (see e.g., Dimitrakopoulos, Economou and Leonardos (2021)).
- *Systems with non-standard or augmented observation structure.* In such systems, the customers observe system features other than or in addition to queue length, like the state of the server or of a random environment etc. (see e.g., Burnetas and Economou (2007), Economou and Manou (2013), and Logothetis and Economou (2023)).

Some other important studies that deal with the influence of information on strategic customer behavior in service systems have been reported in Allon et al. (2011), Armony and Maglaras (2004), Cui and Veeraraghavan (2016), Debo and Veeraraghavan (2016), Guo and Zipkin (2007), Hassin and Roet-Green (2017), Hassin and Snitkovsky (2017), Haviv and Kerner (2007), Hassin and Oz (2016), Hassin and Oz(2018), Ibrahim et al. (2017), Inoue et al. (2023), Kerner (2011), Veeraraghavan and Debo (2009), Veeraraghavan and Debo (2011), Wang et al. (2018), Wang and Hu (2019), and Yu et al. (2018). In these works, the authors examine various important aspects of information influence on customers' behavior in service systems. For detailed summaries and comments, see Hassin (2016), Economou (2021), Economou (2022) and Ibrahim (2018).

3. Hypotheses

Throughout the present study we adopt the usual hypotheses that concern the economic (game-theoretic) analysis of queueing systems. More specifically:

- The queueing systems under study have reached a steady state, in the sense that their various parameters do not change over time. Moreover, the systems have run for a long time so that any effects of the initial conditions have been vanished.

- The customers are assumed to be fully rational, in the sense that they can assess the effects of their actions, taking into account other customers' actions, accurately and effectively.

- The customers are selfish and want to maximize their own utility without bothering about the effect of their actions on other customers or on the administrator of the system.

4. The framework

In the study of strategic customer behavior in queueing systems, the fundamental concepts of classical Game Theory are not directly applicable since there are two significant problems: The first is the fact that the number of customers is infinite, since the potential customers of a service system are infinite. The second is that the customers-players do not make simultaneously their decisions since they arrive sequentially during an infinite time horizon extending to both directions in time. These problems are bypassed by defining analogous concepts and exploiting the homogeneity of the various classes of customers. However, in the present study, to keep the framework as simple as possible, we will assume that all customers are homogeneous.

In the case of homogeneous strategic customers, a Queueing Game among them is specified by the set of their common strategies, S, and from the utility function $U(q, q'|i)$ that specifies the payoff of a customer that uses strategy q then all other customers follow strategy q' and the information $i \in I$ is provided (where I denotes the set of all possible information states-values).

Consider, now, a tagged customer. Given that a strategy q' is used by the population of (the other) customers, a strategy $q^*$of the tagged customer is said to be a best response against q', if q^*maximizes $f(q) = U(q, q'|i)$, for all possible values of the information$i$. The set of best responses against q' is denoted by $BR(q')$. A strategy q^e is said to be a (symmetric) equilibrium, if it is best response against itself, i.e., if $q^e \in BR(q^e)$.

A basic step for the study of strategic customer behavior concerns the computation of the payoff function $U(q, q'|i)$. The fundamental assumption for this computation is that if we consider a tagged customer who follows a strategy q, when all others follow a strategy q', then the tagged customer's strategy does not influence the steady-state behavior of the system. Indeed, the general behavior of the system and the corresponding performance measures are determined by the strategy q' that the other customers follow, since the impact of the tagged customer is negligible. Moreover, it is assumed that the system is in a stochastic steady state. To determine the equilibrium customer strategies in a queueing system, a general methodology is applied, using the following steps:

- Step 1: The steady-state behavior of the system under an arbitrary strategy q' of the population of the customers is studied.

- Step 2: The utility function $U(q, q'|i)$ of a tagged customer that follows strategy q, when all other customers follow strategy q', and the information i is given, is computed.
- Step 3: The best response $BR(q')$ of the tagged customer against an arbitrary strategy, q', of the population of the customers is computed.
- Step 4: All strategies with the property $q^e \in BR(q^e)$ are identified. These are exactly the equilibrium strategies.

A related problem from a social planner's point of view is the maximization of the social welfare per time unit. Then, a socially optimal strategy q_{soc} solves the optimization problem $max_{\, q' \in S} SW(q')$, where $SW(q')$ stands for the social welfare function, when the strategy q' is followed by the population of customers.

In the case of the join-or-balk dilemma, a strategy q, of an arriving customer, corresponds to a vector of join probabilities for the various values of the information, i. Therefore, $q = (q(i): i \in I)$, where $q(i)$ is the join probability for a customer that receives information i upon arrival. Then, the utility function has the form

$$U(q, q'|i) = \big(1 - q(i)\big) \cdot 0 + q(i)(R - CE[W|q', i]), \tag{4.1}$$

where

- R is the service value,
- C is the waiting cost per time unit,
- $q = (q(i): i \in I)$ is the strategy of a tagged customer,
- $q' = (q'(i): i \in I)$ is the strategy of the population of the (other) customers,
- $E[W|q', i]$ is the expected total sojourn time of the tagged customer in the system, if she decides to enter, when the information i is given and the population follows strategy q'.

5. The models

We consider a single server queue with infinite waiting space, where strategic customers arrive according to a Poisson process at rate λ and have independent exponential service times with rate μ, independent of the arrival process. The queue discipline is the First-Come-First-Served (FCFS). We define $\rho = \frac{\lambda}{\mu}$ to be the utilization rate of the model.

Arriving customers at this M/M/1 queue face the dilemma of whether to join or balk with the objective of maximizing their own utilities. Each customer receives a reward of R units upon service completion and accumulates waiting costs at rate C as long as she stays in the system. We define $\nu = \frac{R\mu}{C}$ to be the normalized service value, which corresponds to the ratio of the service value over the mean cost of a service time.

The operational and economic parameters of the system are assumed to be common knowledge for all customers. This is a reasonable assumption when the population of customers visits the system recurrently.

In the classical Edelson and Hildebrand (1975) unobservable model, the 4-step procedure for identifying the equilibrium strategies proceeds as follows:

- Step 1: Since the customers receive no information in this case, a customer's strategy is just her join probability q. Under an arbitrary strategy, q', the system behaves as an M/M/1 queue with arrival rate $\lambda q'$ and service rate μ. The state of the system is represented by a continuous-time Markov chain $N(t)$ which records the number of customers in the system. Its transition matrix is given as

$$Q = \begin{pmatrix} -\lambda q' & \lambda q' & 0 & 0 & 0 & \cdots \\ \mu & -(\lambda q' + \mu) & \lambda q' & 0 & 0 & \cdots \\ 0 & \mu & -(\lambda q' + \mu) & \lambda q' & 0 & \cdots \\ 0 & 0 & \mu & -(\lambda q' + \mu) & \lambda q' & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}.$$

- Step 2: The waiting time of an arriving customer at an M/M/1 queue with arrival rate $\lambda q'$ and service rate μ is known to be exponentially distributed with rate $\mu - \lambda q'$. Hence, the expected total sojourn time, if she decides to join is $\frac{1}{\mu - \lambda q'}$. Therefore, the utility function, given by (4.1), assumes the form

$$U(q, q') = (1 - q) \cdot 0 + q \cdot \left(R - \frac{C}{\mu - \lambda q'}\right). \qquad (5.1)$$

- Step 3: Since $U(q, q')$ is a linear function of$q$, its maximum occurs at $q = 1$ (respectively at $q = 0$ or for every $q \in [0,1]$), if $R - \frac{C}{\mu - \lambda q'} > 0$ (respectively if $R - \frac{C}{\mu - \lambda q'} < 0$ or $R - \frac{C}{\mu - \lambda q'} = 0$).

  Therefore, the set of best responses, $BR(q')$, of a tagged customer against an arbitrary strategy, q', of the population of the customers is given as

$$BR(q') = \begin{cases} \{0\}, & \text{if} q' > \overline{q_e}, \\ [0,1], & \text{if} q' = \overline{q_e}, \\ \{1\}, & \text{if} q' < \overline{q_e}, \end{cases}$$

  where

$$\overline{q_e} = \frac{1}{\lambda}\left(\mu - \frac{C}{R}\right).$$

  is the root of $R - \frac{C}{\mu - \lambda q'} = 0$.

- Step 4: We can now proceed to the computation of the equilibrium strategies:

The strategy of `always balk' ($q_e = 0$) is equilibrium strategy, if and only if $0 \in BR(0)$, i.e., $0 \geq \overline{q_e}$, which reduces to $R \leq \frac{C}{\mu}$.

A strategy $q_e \in (0,1)$ is equilibrium strategy, if and only if $q_e \in BR(q_e)$, i.e., $q_e = \overline{q_e}$, which reduces to $q_e = \frac{1}{\lambda}\left(\mu - \frac{C}{R}\right)$. This is valid as far as $\overline{q_e} \in (0,1)$, which occurs if and only if $\frac{C}{\mu} < R < \frac{C}{\mu-\lambda}$.

Finally, the `always join' ($q_e = 1$) is equilibrium strategy, if and only if $1 \in BR(1)$, i.e., $1 \leq \overline{q_e}$, which reduces to $R \geq \frac{C}{\mu-\lambda}$.

In conclusion, for the unobservable model, a unique equilibrium strategy q_e exists, given by the formula

$$q_e = \begin{cases} 0, & \text{if } R \leq \frac{C}{\mu}, \\ \frac{1}{\lambda}\left(\mu - \frac{C}{R}\right), & \text{if } \frac{C}{\mu} \leq R \leq \frac{C}{\mu-\lambda}, \\ 1, & R \geq \frac{C}{\mu-\lambda}. \end{cases}$$

In what follows, we will present various models where the information that is provided to the customers concerns the decisions of previous customers. For each model, we will describe the stochastic processes for steps 1 and 2 of the 4-step process. This constitutes the queueing part of the problems, the part that is treated in the present paper. Steps 3 and 4 which constitute the game-theoretical part of the study will be presented in future work. However, some preliminary results are shown in Section 6 (a summary of the results in Economou (2024)).

5.1 The detailed *N*-Bernoulli information scheme

Under the detailed N-Bernoulli information scheme, the arriving customers are informed about the join-or-balk decisions of the last N previous arrivals. For example, for $N = 3$, the information $\mathrm{i} = (0,0,1)$ means that the last two arrivals balked, whereas the arrival just before them entered.

A customer's strategy in this case is a vector $q = \left(q_{i_1,i_2,\ldots,i_N} : (i_1, i_2, \ldots, i_N) \in \{0,1\}^N\right)$, where $q_{i_1,i_2,\ldots,i_N}$ is the join probability for a customer who receives the information $I(t) = (i_1, i_2, \ldots, i_N)$ regarding the decisions of the last Narrivals.

Under a given strategy q of the population, the process $\{(N(t), I(t))\}$, where $N(t)$ is the number of customers in the system at time t and $I(t)$ the information value at time t is a QBD process with transition rate matrix

$$Q = \begin{pmatrix} B_{0,0} & B_{0,1} & 0 & 0 & 0 & \cdots \\ B_{1,0} & A_1 & A_0 & 0 & 0 & \cdots \\ 0 & A_2 & A_1 & A_0 & 0 & \cdots \\ 0 & 0 & A_2 & A_1 & A_0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}. \quad (5.2)$$

Each block is of size $2^N \times 2^N$, where N is the number of previous customers for whom the join-or-balk information is provided.

For example, for the simplest case, where $N = 1$, we have that $\{(N(t), I(t))\}$ is a QBD process, where $\{N(t)\}$ corresponds to the level and $\{I(t)\}$ corresponds to the phase of the process. Its transition diagram under a general strategy $q = (q_0, q_1)$ is presented in Figure 1.

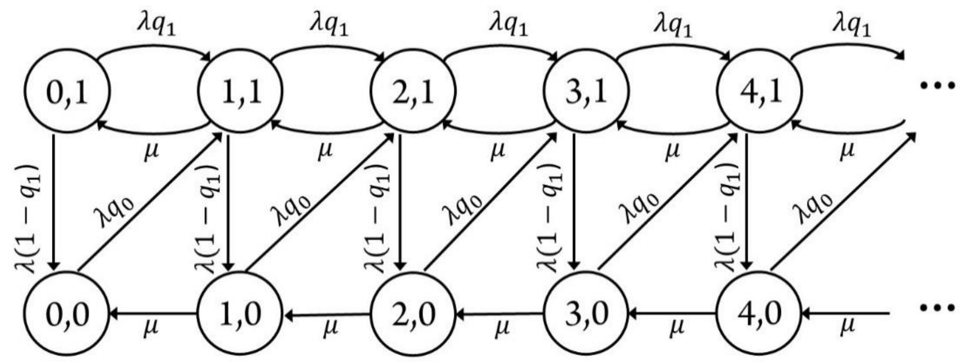


Figure 1: Transition diagram of $\{(N(t), I(t))\}$ for the detailed 1-Bernoulli information scheme under a given population strategy (q_0, q_1).

The QBD blocks that appear in matrix (4.2) for $N = 1$ are as follows:

$$A_0 = \begin{pmatrix} 0 & \lambda q_0 \\ 0 & \lambda q_1 \end{pmatrix}, A_1 = \begin{pmatrix} -(\lambda q_0 + \mu) & 0 \\ \lambda(1 - q_1) & -(\lambda + \mu) \end{pmatrix},$$

$$A_2 = \mu I, B_{0,0} = A_0 + A_1, B_{0,1} = A_0, B_{1,0} = A_2.$$

For $N = 2$ the phase-space has now 4 elements and a general strategy is given as $q = (q_{00}, q_{01}, q_{10}, q_{11})$. Under this strategy, the QBD blocks that appear in matrix (5.2) assume the form

$$A_0 = \begin{pmatrix} 0 & 0 & \lambda q_{00} & 0 \\ 0 & 0 & \lambda q_{01} & 0 \\ 0 & 0 & 0 & \lambda q_{10} \\ 0 & 0 & 0 & \lambda q_{11} \end{pmatrix},$$

$$A_1 = \begin{pmatrix} -(\lambda q_{00} + \mu) & 0 & 0 & 0 \\ \lambda(1 - q_{01}) & -(\lambda + \mu) & 0 & 0 \\ 0 & \lambda(1 - q_{10}) & -(\lambda + \mu) & 0 \\ 0 & \lambda(1 - q_{11}) & 0 & -(\lambda + \mu) \end{pmatrix},$$

$$A_2 = \mu I, B_{0,0} = A_0 + A_1, B_{0,1} = A_0, B_{1,0} = A_2.$$

5.2 The geometric run-of-1s information scheme

Under the geometric run-of-1s information scheme, the arriving customers are informed about the number of customers joining since the last balking customer. For example, the information $i = 5$ means that the last 5 arrivals entered whereas the arrival just before them did not enter.

A customer's strategy is a sequence $q = (q_0, q_1, q_2, \ldots)$, where q_i is the join probability for a customer who receives the information $I(t) = i$.

Under a given strategy of this form, the process $\{(N(t), I(t))\}$ which records the current number of customers and the corresponding information value is again a QBD process with transition rate matrix given by (5.2), where the QBD blocks are of infinite size, since $I(t)$ takes values in $\mathcal{I} = \{0,1,2,\ldots\}$. The QBD blocks under an arbitrary strategy $q = (q_0, q_1, q_2, \ldots)$ are the following:

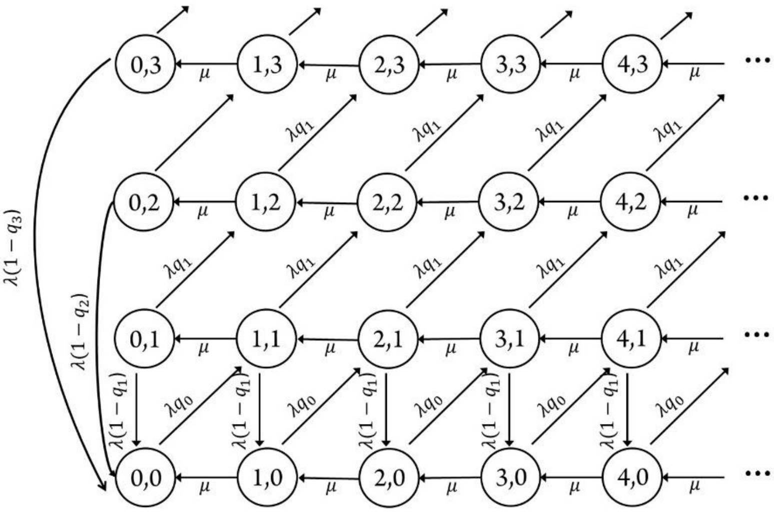
$$A_0 = \begin{pmatrix} 0 & \lambda q_0 & 0 & 0 & 0 & \cdots \\ 0 & 0 & \lambda q_1 & 0 & 0 & \cdots \\ 0 & 0 & 0 & \lambda q_2 & 0 & \cdots \\ 0 & 0 & 0 & 0 & \lambda q_3 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix},$$

$$A_1 = \begin{pmatrix} -(\lambda q_0 + \mu) & 0 & 0 & 0 & 0 & \cdots \\ \lambda(1-q_1) & -(\lambda+\mu) & 0 & 0 & 0 & \cdots \\ \lambda(1-q_2) & 0 & -(\lambda+\mu) & 0 & 0 & \cdots \\ \lambda(1-q_3) & 0 & 0 & -(\lambda+\mu) & 0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix},$$

$$A_2 = \mu I, B_{0,0} = A_0 + A_1, B_{0,1} = A_0, B_{1,0} = A_2.$$

The transition diagram is presented in Figure 2. To make the transitions clearer we have shown the curved arrows that correspond to transitions of the form $(n,i) \to (n,0)$, with corresponding rates $\lambda(1-q_i)$ only for $n = 0$ in the diagram. For $n \geq 1$ the same transitions are possible, but we have omitted the curved arrows from the diagram.

Figure 2: Transition diagram of $\{(N(t), I(t))\}$ for the geometric run-of-1s information scheme under a given population strategy $q = (q_0, q_1, \ldots)$.



5.3 The geometric run-of-0s information scheme

This is the dual of the geometric run-of-1s information scheme where the customers are informed about the number of balking customers since the last joining customer. A customer's strategy is again a sequence $q = (q_0, q_1, q_2, \ldots)$, where q_i is the join probability for a customer that receives information $I(t) = i$. Under a given arbitrary strategy, the joint process of the number of present customers and the corresponding information value is a QBD with a similar structure as in the geometric run-of-1s case.

5.4 Truncated geometric information scheme

In the truncated-at-N geometric run-of-1s information scheme the customers are informed about the number of joining customers since the last balking customer being 0 or 1 or 2 or $\ldots N$ or `above N'. This model is a truncated version of the geometric run-of-1s information scheme. Although it seems a bit artificial, it has the advantage that it corresponds to a QBD process with finite phase-space as opposed to the original geometric run-of-1s scheme where the phase-space is infinite.

Similarly, the truncated-at-N geometric run-of-0s information scheme is a truncated version of the original geometric run-of-0s information scheme, with a corresponding QBD process that has finite phase-space.

5.5 The binomial information scheme

Under the binomial information scheme, the customers are informed about the number of joining customers among the last N more recent arrivals. For example, for $N =$

7, the information $i = 3$ means that, among the last 7 more recent arrivals, 3 of them entered. A customer's strategy is a vector $q = (q_0, q_1, q_2, \dots, q_N)$, where q_i is the join probability for a customer who receives information i.Under a given strategy of this form, the process $\{(N(t), I(t))\}$ which records the current number of customers, and the corresponding information is not Markovian. Therefore, to study this information case, we use the QBD process that we defined for the detailed N-Bernoulli information scheme and proceed with conditioning arguments for the various computations.

6. Equilibrium customer strategies for the detailed 1-Bernoulli information scheme

We now consider the case where each customer is informed about the decision of the last arrival before her, being join (1) or balk (0), and then makes her own decision. This corresponds to the detailed 1-Bernoulli information scheme. We will refer to this model as the last-customer's-decision (lcd) model when we compare it with other models that have been reported in the literature as the unobservable (un) and the observable (obs) models.

Consider, now, a tagged arriving customer and let S be her sojourn time in the system. Moreover, let $N^-$and I^- be the number of customers in the system and the decision of the last arrival before her. Due to the Poisson arrivals (PASTA property) we have that the joint distribution of (N^-, I^-) coincides with the steady-state distribution of $\{(N(t), I(t))\}$ in continuous time. Therefore, the conditional probability that the tagged customer will see n customers in the system, given that the last arrival before her made the decision i is

$$\Pr[Q^- = n | I^- = i] = \frac{p(n,i)}{p_I(i)}, \quad n \geq 0, \quad i = 0,1, \tag{6.1}$$

where $(p(n,i): n \geq 0, i = 0,1)$ is the steady-state distribution of $\{(N(t), I(t))\}$ and $p_I(n) = \sum_{n=0}^{\infty} p(n,i)\, , i = 0,1$. Hence, we can easily see that

$$E[S | I^- = i] = \sum_{n=0}^{\infty} \frac{n+1}{\mu} \cdot \frac{p(n,i)}{p_I(i)} = \frac{E[Q | I = i]+1}{\mu} = \frac{P_i'(1)/p_I(i)+1}{\mu}, \tag{6.2}$$

where $P_i(z) = \sum_{n=0}^{\infty} p(n,i) z^n,\ i = 0,1$. Using the balance equations for themodel and following the generating function approach (see Economou (2024)) yields

$$p_I(0) = \frac{1-q_1}{1-q_1+q_0} \text{ and } p_I(1) = \frac{q_0}{1-q_1+q_0}, \tag{6.3}$$

and

$$P_0(z) = \frac{1-q_1}{1-q_1+q_0} \cdot \frac{\rho q_0+\rho+1-\rho_2-\rho q_1 z}{\rho q_0+\rho+1-\rho_2-\rho q_1} \cdot \frac{1-\rho_2}{1-\rho_2 z}, \tag{6.4}$$

$$P_1(z) = \frac{q_0}{1-q_1+q_0} \cdot \frac{1+(\rho-\rho_2)z}{1+\rho-\rho_2} \cdot \frac{1-\rho_2}{1-\rho_2 z}, \tag{6.5}$$

where $\rho_2 = \rho_2(q_0, q_1)$ is given by

$$\rho_2 = \rho_2(q_0, q_1) = \frac{\rho q_0 + \rho + 1 - \sqrt{(\rho q_0 + \rho + 1)^2 - 4(\rho^2 q_0 + \rho q_1)}}{2}. \quad (6.6)$$

Let $S_i(q_0, q_1)$ be the net benefit of a tagged arriving customer who sees $I^- = i$ upon arrival and decides to join, given that the population of customers follows a strategy (q_0, q_1). Then, we have that

$$S_i(q_0, q_1) = R - \frac{C}{\mu} E[S | I^- = i]. \quad (6.7)$$

Using (6.7), (6.2) and evaluating, at $z = 1$, the derivatives of $P_0(z), P_1(z)$, given by (6.4) and (6.5), we derive the following explicit formulas for the quantities $S_i(q_0, q_1), i = 0,1$:

$$S_0(q_0, q_1) = R - \frac{C}{\mu}\left(\frac{1}{1-\rho_2} - \frac{\rho\rho_2 q_1}{\rho^2 q_0 + \rho(1-\rho_2) q_1}\right), \quad (6.8)$$

$$S_1(q_0, q_1) = R - \frac{C}{\mu}\left(\frac{1}{1-\rho_2} + \frac{\rho(\rho-\rho_2) q_0 + \rho q_1 - \rho_2}{\rho(\rho-\rho_2) q_0 + \rho q_1}\right). \quad (6.9)$$

Using the formulas (6.3) we can easily derive the throughput of the system under a given customer strategy (q_0, q_1). In conjunction with (6.8) and (6.9), we can also obtain the social welfare per time unit generated by the system.

The throughput generated from a customer strategy (q_0, q_1) is given by

$$TH^{lcd}(q_0, q_1) = p_I(0)\lambda q_0 + p_I(1)\lambda q_1 = \frac{\lambda q_0}{1 - q_1 + q_0} \quad (5.10)$$

and the corresponding welfare is

$$SW^{lcd}(q_0, q_1) = p_I(0)\lambda q_0 S_0(q_0, q_1) + p_I(1)\lambda q_1 S_1(q_0, q_1)$$
$$= \frac{\lambda q_0 (1-q_1) S_0(q_0, q_1) + \lambda q_0 q_1 S_1(q_0, q_1)}{1 - q_1 + q_0}. \quad (5.11)$$

Formulas (6.8) and (6.9) show that

$$S_0(q_0, q_1) > S_1(q_0, q_1). \quad (5.12)$$

This inequality implies that the only possible forms for an equilibrium strategy are $(0,0), (q_0^*, 0), (1,0), (1, q_1^*)$ and $(1,1)$, with $q_0^*, q_1^* \in (0,1)$. Indeed, a strategy (q_0^*, q_1^*) is equilibrium if only if $(q_0^*, q_1^*) \in BR(q_0^*, q_1^*)$, so we have the following cases:

1. $(0,0)$ is equilibrium strategy if and only if $S_0(0,0) \leq 0$.
2. $(q_0^*, 0)$ with $q_0^* \in (0,1)$ is equilibrium strategy if and only if $S_0(q_0^*, 0) = 0$.
3. $(1,0)$is equilibrium strategy if and only if $S_1(1,0) \leq 0 \leq S_0(1,0)$.
4. $(1, q_1^*)$ with $q_1^* \in (0,1)$ is equilibrium strategy if and only if $S_1(1, q_1^*) = 0$.
5. $(1,1)$ is equilibrium strategy if and only if $S_1(1,1) \geq 0$.

Therefore, the key for the computation of the equilibrium strategies for a specific instance of the model (for given parameters λ, μ, R and C) is the calculation of the quantities $S_0(0,0), S_0(1,0)$, $S_1(1,0)$, $S_1(1,1)$ and the solution of the equations $S_0(x, 0) = 0$ and $S_1(1, x) = 0$ in (0,1). To this end we can use the formulas (6.8) and (6.9). Considering the above 5 cases regarding the form of equilibrium strategies and solving with respect to the normalized service value $\nu = \frac{R\mu}{C}$, we characterize the equilibrium strategy, as the normalized service value ν assumes values from 0 to ∞.

Theorem 6.1 An equilibrium customer strategy for the lcd model exists and is unique for any values of the underlying parameters. If $\rho < 1$, then we have the following cases regarding the equilibrium strategy $\left(q_0^{lcd-e}, q_1^{lcd-e}\right)$:

1. $\nu \in [0,1]$. Then $\left(q_0^{lcd-e}, q_1^{lcd-e}\right) = (0,0)$.
2. $\nu \in \left(1, \frac{2}{1-2\rho+\sqrt{1+4\rho}}\right)$. Then $\left(q_0^{lcd-e}, q_1^{lcd-e}\right) = (q_0^*, 0)$ with
$$q_0^* = \frac{1}{\frac{1}{\nu}+\rho-1} - \frac{1}{\nu\rho}. \tag{6.13}$$
3. $\nu \in \left[\frac{2}{1-2\rho+\sqrt{1+4\rho}}, \frac{5\rho+1-(\rho+1)\sqrt{1+4\rho}}{3\rho-\rho\sqrt{1+4\rho}}\right]$. Then $\left(q_0^{lcd-e}, q_1^{lcd-e}\right) = (1,0)$.
4. $\nu \in \left(\frac{5\rho+1-(\rho+1)\sqrt{1+4\rho}}{3\rho-\rho\sqrt{1+4\rho}}, \frac{1}{1-\rho}\right)$. Then $\left(q_0^{lcd-e}, q_1^{lcd-e}\right) = (1, q_1^*)$ with
$$q_1^* = \frac{3-\rho-\frac{2}{\nu-1}+(1-\rho)\sqrt{1+\frac{4}{\rho(\nu-1)}}}{2}. \tag{6.14}$$
5. $\nu \in \left[\frac{1}{1-\rho}, \infty\right)$. Then $\left(q_0^{lcd-e}, q_1^{lcd-e}\right) = (1,1)$.

Now, we can use the formulas (6.10) and (6.11) to obtain the equilibrium throughput and the equilibrium social welfare as ν increases from 0 to ∞ (for details see Economou (2024)).

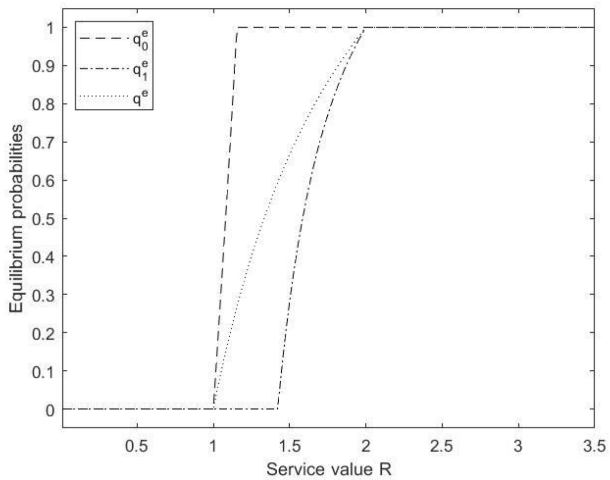
7. Comparison with other information structures

In this section we present some numerical results concerning the comparison of the lcd model with the un (Edelson and Hildebrand (1975)) and obs (Naor (1969)) models. Let q^{un-e} be the equilibrium probability of the un model (see e.g., Hassin and Haviv (2003) Table 3.1) and TH^{un-e}, SW^{un-e} be the corresponding equilibrium throughput, equilibrium social welfare. Moreover, we denote by TH^{obs-e} and SW^{obs-e} the throughput and the social welfare of the obs model when customers enter according to Naor's individually optimal threshold.

To present several numerical results concerning the comparison of the various models, we will consider a numerical experiment with parameters $\lambda = 0.5$, $\mu = 1$, $C = 1$ and $R \in [0,3.5]$, which we will refer to as the `standard numerical scenario'. Note that in this scenario we have that $\rho = 0.5$, and $\nu \in [0,3.5]$. This numerical scenario is typical. Indeed, we have considered a large number of other parameter values, and the various figures have similar shapes leading to the same findings and interpretations.

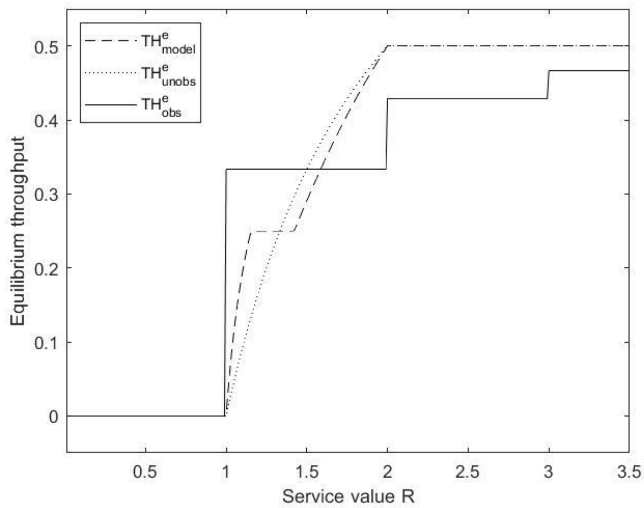
In Figure 3 we present the graphs of the equilibrium join probabilities as functions of the service value for the lcd and the un models.

Figure 3: Equilibrium join probabilities for $\lambda = 0.5$, $\mu = 1$, $C = 1$, $R \in [0, 3.5]$.



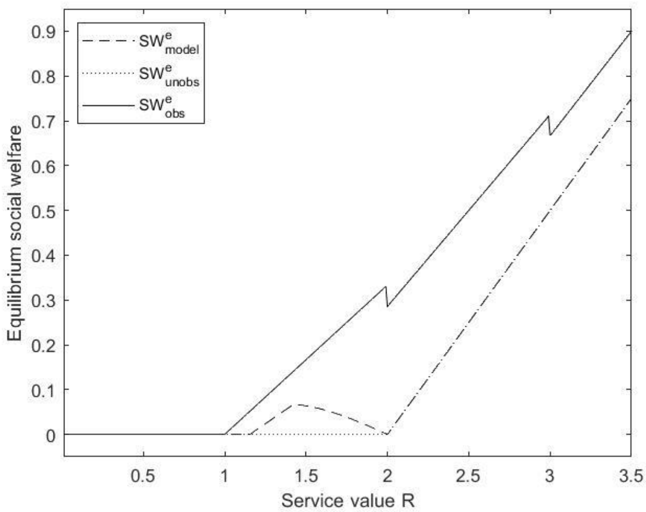
In Figure 4, we show the graphs of the equilibrium social welfare functions, as the service value varies, for the standard numerical scenario, for the un, lcd and obs models.

Figure 4: Equilibrium social welfare functions for $\lambda = 0.5$, $\mu = 1$, $C = 1$, $R \in [0, 3.5]$.



In Figure 5, we see the graphs of the equilibrium throughput functions for the standard numerical scenario.

Figure 5: Equilibrium throughput functions for $\lambda = 0.5$, $\mu = 1$, $C = 1$, $R \in [0, 3.5]$



The main findings from these graphs about the equilibrium probabilities (EP), the social welfare functions (SW) and the throughput functions (TH), that have consistently observed in all numerical scenarios that we have studied, are listed below:

- **EP1** $q_0^{lcd-e} \geq q^{un-e} \geq q_1^{lcd-e}$. This universal inequality shows that the join probability of the un model is always between the two join probabilities for the lcd model.
- **SW1** $SW^{obs-e} \geq SW^{lcd-e} \geq SW^{un-e}$. This universal inequality shows that the observable model outperforms the lcd model which in turn outperforms the un model in terms of social welfare.
- **SW2** The difference $SW^{lcd-e}(\nu) - SW^{un-e}(\nu)$ is a unimodal function of ν which attains its maximum at $\frac{5\rho+1-(\rho+1)\sqrt{1+4\rho}}{3\rho-\rho\sqrt{1+4\rho}}$. This value constitutes the boundary between Cases 3 and 4 of Theorem 6.1, that is, it is the value where the equilibrium strategy changes from (1,0) to $(1, q_1^*)$ with $q_1^* \in (0,1)$.
- **TH1** $TH^{obs-e}(\nu) > TH^{lcd-e}(\nu) > TH^{un-e}(\nu)$, for low values of ν (but greater than 1).
- **TH2** $TH^{obs-e}(\nu) < TH^{lcd-e}(\nu) < TH^{un-e}(\nu)$, for high values of ν (but smaller than $\frac{1}{1-\rho}$).
- **TH3** $TH^{obs-e}(\nu) < TH^{lcd-e}(\nu) = TH^{un-e}(\nu)$, for $\nu \geq \frac{1}{1-\rho}$.
- **TH4** The functions $TH^{lcd-e}(\nu)$ and $TH^{un-e}(\nu)$ cross only once in $\left(1, \frac{1}{1-\rho}\right)$. More concretely, there exists ν^* such that $TH^{lcd-e}(\nu) > TH^{un-e}(\nu)$, for $\nu \in (1, \nu^*)$, whereas $TH^{lcd-e}(\nu) < TH^{un-e}(\nu)$, for $\nu \in \left(\nu^*, \frac{1}{1-\rho}\right)$.

8. Results

In this paper, we introduce several models for the communication of recent customers' decisions to the strategic customers of a service system. Moreover, we present some preliminary results for the customer strategic behavior in these models and compare it with the corresponding behavior in the classical observable and unobservable models.

For the join-or-balk dilemma of strategic customers at an M/M/1 queue, it turns out that this new type of information smooths the effective arrival process and is beneficial for the social welfare in comparison to the unobservable model, in particular for intermediate values of the service reward.

9. Conclusions

In systems where the information about the queue length cannot be communicated to the customers, the provision of information about the last customer's decision has been shown to be advantageous. This was outlined in this paper and presented in more detail in Economou (2024).

The present paper suggests a variety of models that are based on the same idea, i.e. informing customers regarding other customers' decisions. The research effort should be continued to clarify in more depth the implications of such types of information. In particular, the following research questions seem important for extending the results:

- What is the effect of informing customers about recent customers' decisions on the customers' utility, administrator's profit and social welfare? Who benefits and who loses from such type of information?
- Can the analysis be extended into more complex queueing models, for example models with many servers and/or batch arrivals and departures?
- How can the analysis be carried out in the case of heterogeneous customers (regarding their service value and waiting costs)?

References

Allon, G., Bassamboo, A. & Gurvich, I. (2011). We will be right with you": Managing customer expectations with vague promises and cheap talk. *Operations Research, 59*, 1382-1394.

Armony, M., & Maglaras, C. (2004). Contact centers with a call-back option and real-time delay information. *Operations Research, 52*, 527-545.

Burnetas, A., & Economou, A. (2007) Equilibrium customer strategies in a single server Markovian queue with setup times. *Queueing Systems56*, 213-228.

Burnetas, A., Economou, A., & Vasiliadis, G. (2017). Strategic behavior in a queueing system with delayed observations. *Queueing Systems, 86*, 389-418.

Chen, H., & Frank, M. (2004). Monopoly pricing when customers queue. *IIE Transactions36*, 569-581.

Cui, S., & Veeraraghavan, S. (2016). Blind Queues: The impact of consumer beliefs on revenues and congestion. *Management Science, 62*, 3656-3672.

Debo, L. & Veeraraghavan, S. (2014) Equilibrium in queues under unknown service times and service value. *Operations Research62*, 38-57.

Dimitrakopoulos, Y., Economou, A., & Leonardos, S. (2021). Strategic customer behavior in a queueing system with alternating information structure. *European Journal of Operational Research, 291*, 1024-1040.

Economou, A. (2021). The impact of information structure on strategic behavior in queueing systems. In Anisimov, V. and Limnios, N. Eds. *Queueing Theory 2, Advanced Trends*, Series `Mathematics and Statistics', Sciences, ISTE & J. Wiley, London.

Economou, A. (2022). How much information should be given to the strategic customers of a queueing system? *Queueing Systems, 100*(3-4), 421-423.

Economou, A. (2024). The impact of information about last customer's decision on the join-or-balk dilemma in a queueing system. *Annals of Operations Research*, forthcoming.

Economou, A., & Grigoriou, M. (2015). Strategic balking behavior in a queueing system with a mixed observation structure. *Proceedings of the 10th Conference on Stochastic Models of Manufacturing and Service Operations* (SMMSO 2015) University of Thessaly Press, Volos, 51-58.

Economou, A., & Kanta, S. (2008). Optimal balking strategies and pricing for the single server Markovian queue with compartmented waiting space. *Queueing Systems, 59*, 237-269.

Economou, A., & Manou, A. (2013). Equilibrium balking strategies for a clearing queueing system in alternating environment. *Annals of Operations Research, 208*, 489-514.

Edelson, N.M., & Hildebrand, K. (1975). Congestion tolls for Poisson queueing processes. *Econometrica, 43*, 81-92.

Guo, P., & Zipkin, P. (2007). Analysis and comparison of queues with different levels of delay information. *Management Science, 53*, 962-970.

Guo, P., & Zipkin, P. (2009). The effects of the availability of waiting-time information on a balking queue. *European Journal of Operational Research, 198*, 199-209.

Hassin, R. (1986). Consumer information in markets with random products quality: The case of queues and balking. *Econometrica, 54*, 1185-1195.

Hassin, R. (2016). *Rational Queueing.* CRC Press, Taylor and Francis Group, Boca Raton.

Hassin, R., & Haviv, M. (2003). *To Queue or Not to Queue: Equilibrium Behavior in Queueing Systems.* Kluwer Academic Publishers, Boston.

Hassin, R., & Koshman, A. (2017). Profit maximization in the M/M/1 queue. *Operations Research Letters, 45*, 436-441.

Hassin, R., & Roet-Green, R. (2017). The impact of inspection cost on equilibrium, revenue, and social welfare in a single-server queue. *Operations Research, 65*, 804-820.

Hassin, R., & Roet Green, R. (2020). On queue-length information when customers travel to a queue. *Manufacturing & Service Operations Management, 23* (4), 989-1004.

Hassin, R., & Snitkovsky, R. (2020). Social and monopoly optimization in observable queues. *Operations Research, 68*(4), 1178-1198.

Haviv, M., & Kerner, Y. (2007). On balking from an empty queue. *Queueing Systems, 55* (4), 239-249.

Haviv, M., & Oz, B. (2016). Regulating an observable M/M/1 queue. *Operations Research Letters, 44* (2), 196-198.

Haviv, M., & Oz, B. (2018). Self-regulation of an unobservable queue. *Management Science, 64* (5), 2380--2389.

Hu, M., Li, Y., & Wang, J. (2018). Efficient ignorance: Information heterogeneity in a queue. *Management Science, 64*, 2650-2671.

Ibrahim, R. (2018). Sharing delay information in service systems: a literature survey. *Queueing Systems, 89*, 49-79.

Ibrahim, R., Armony, M., & Bassamboo, A. (2017). Does the past predict the future? The case of delay announcements in service systems. *Management Science, 63*, 1762-1780.

Inoue, Y., Ravner, L., & Mandjes, M. (2023). Estimating customer impatience in a service system with unobserved balking. *Stochastic Systems, 13* (2), 181-210.

Kerner, Y. (2011). Equilibrium joining probabilities for an M/G/1 queue. *Games and Economic Behavior, 71* (2), 521-526.

Logothetis, D., & Economou, A. (2023). The impact of information on transportation systems with strategic customers. *Production and Operations Management, 32*, 2189-2206.

Naor, P. (1969). The regulation of queue size by levying tolls. *Econometrica, 37*, 15-24.

Nelson, R. (1995). *Probability, Stochastic Processes and Queueing Theory: The Mathematics of Computer Performance Modeling.* Springer.

Stidham, S. Jr. (2009). *Optimal Design of Queueing Systems.* CRC Press, Taylor and Francis Group, Boca Raton.

Veeraraghavan, S., & Debo, L. (2009). Joining longer queues: Information externalities in queue choice. *Manufacturing & Service Operations Management, 11*, 543-562.

Veeraraghavan, S.K., & Debo, L.G. (2011). Herding in queues with waiting costs: Rationality and regret. *Manufacturing & Service Operations Management, 13*, 329-346.

Wang, J., Cui, S., & Wang, Z. (2018). Equilibrium strategies in M/M/1 priority queues with balking. *Production and Operations Management, 28*, 43-62.

Wang, J. & Hu, M. (2020). Efficient inaccuracy: User-generated information sharing in a queue. *Management Science, 66* (10), 4648-4666.

Yu, Q., Allon, G., Bassamboo, A., & Iravani, S. (2018). Managing customer expectations and priorities in service systems. *Management Science, 64*, 3942-3970.